Mining Time-constrained Sequential Patterns with CP

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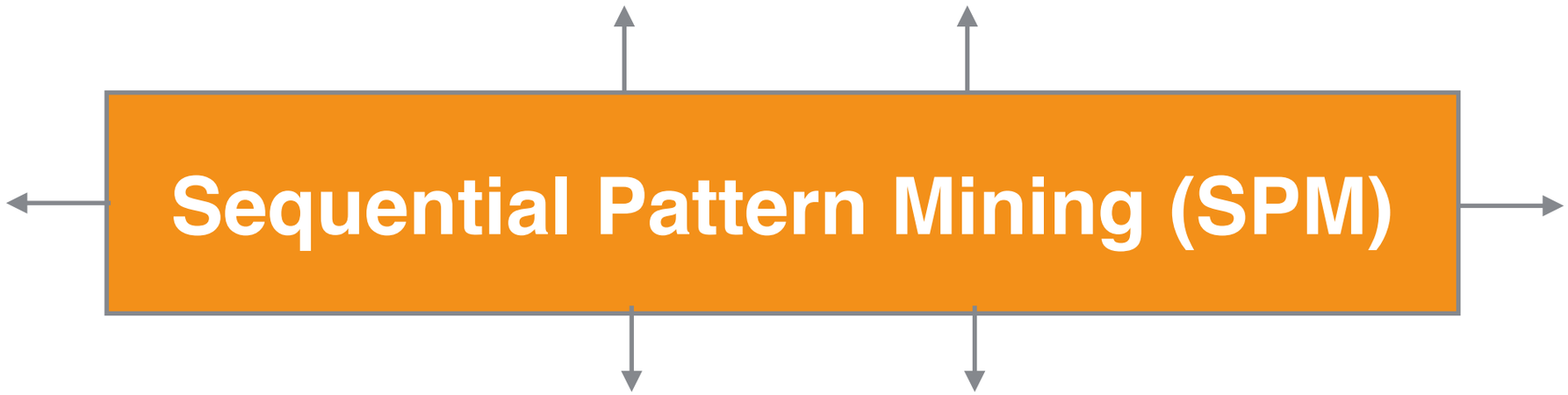
General Overview



General Overview

Telecom

- Network analysis
- People behavior
- Inter-Connection

Sequential Pattern Mining (SPM)

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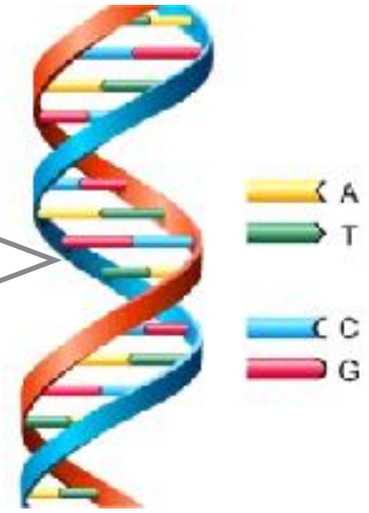
- Network analysis
- People behavior
- Inter-Connection

Bioinformatics

- DNA Analysis
- Disease Analysis
- Gene Analysis



Sequential Pattern Mining (SPM)

General Overview

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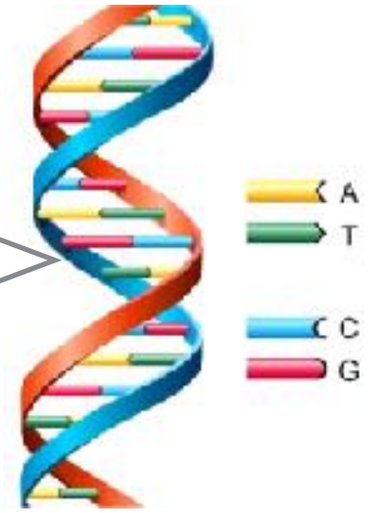


Sequential Pattern Mining (SPM)

Recommendation

- Purchase analysis
- web usage mining
- Comments mining

SPM Problem

Client1	Milk	Coffee	Sugar	Coffee	Sugar
Client2	Coffee	Milk	Coffee	Sugar	
Client3	Milk	Coffee			
Client4	Coffee	Sugar	Egg		

Sequence Database (SDB)

SPM Problem

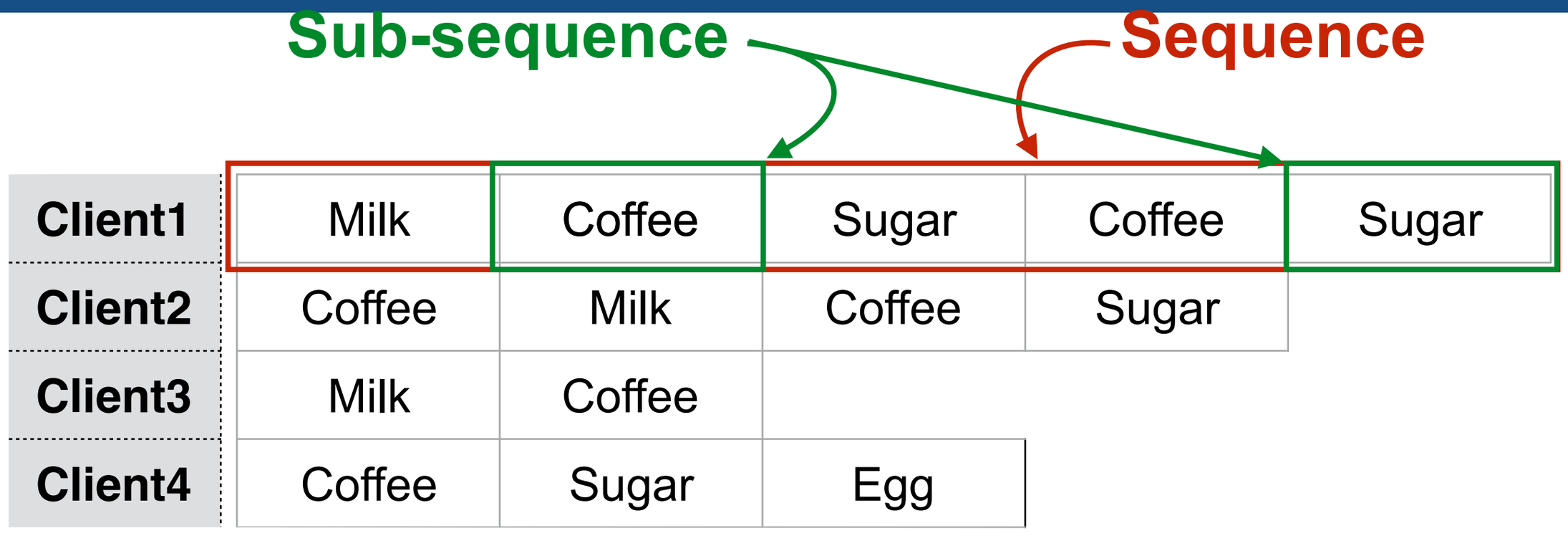
Sequence

Client1	Milk	Coffee	Sugar	Coffee	Sugar
Client2	Coffee	Milk	Coffee	Sugar	
Client3	Milk	Coffee			
Client4	Coffee	Sugar	Egg		

Sequence Database (SDB)

- Sequence : < Milk Coffee Sugar Coffee Sugar>

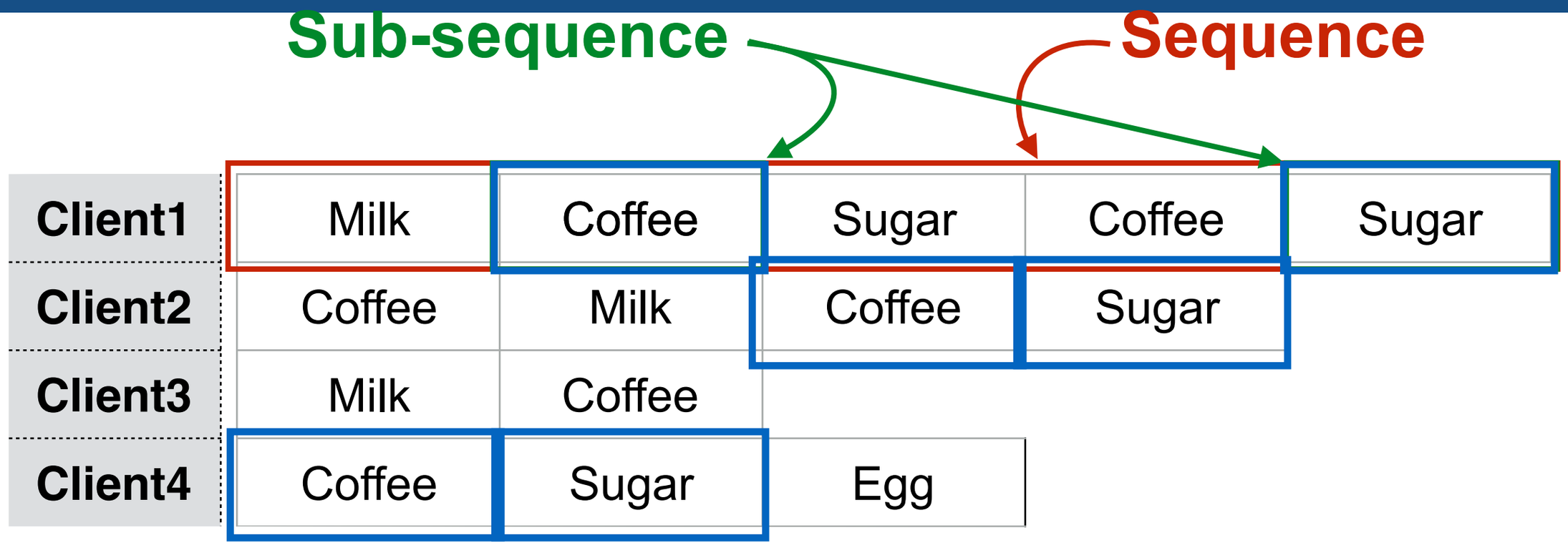
SPM Problem



Sequence Database (SDB)

- Sequence : < Milk Coffee Sugar Coffee Sugar>
- Sub-sequence : <Coffee Sugar>

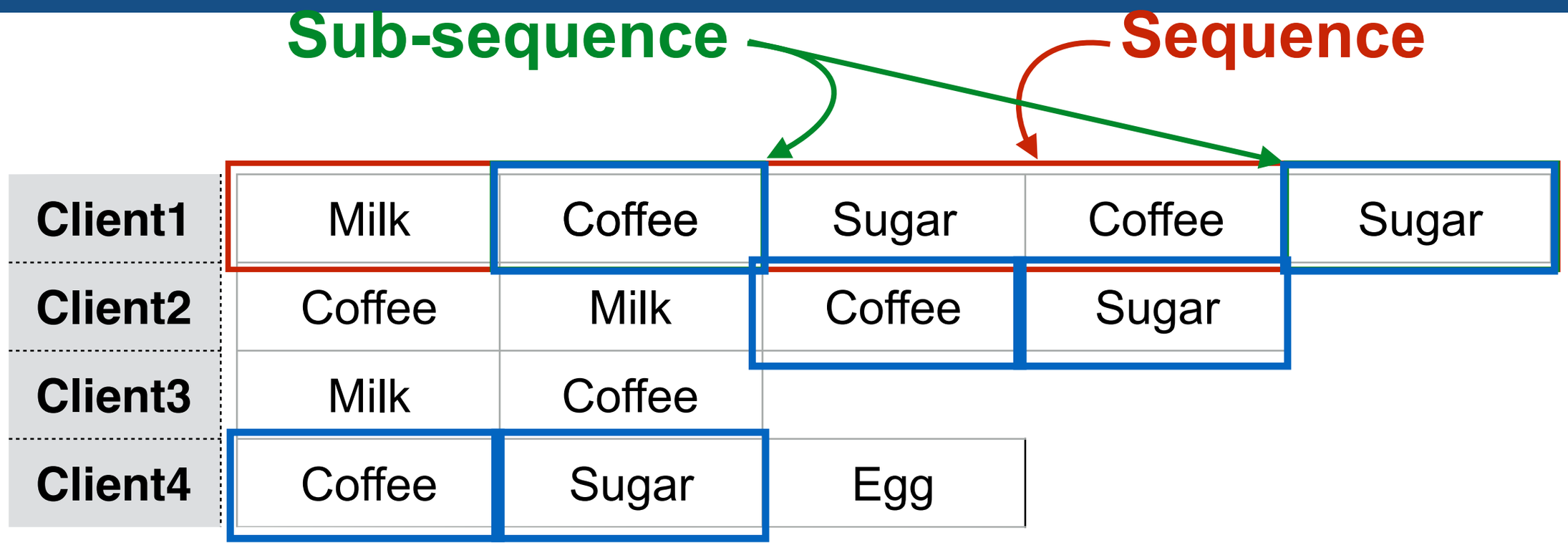
SPM Problem



Sequence Database (SDB)

- Sequence : < Milk Coffee Sugar Coffee Sugar>
- Sub-sequence : <Coffee Sugar>
- Support (<Coffee Sugar>) = 3

SPM Problem

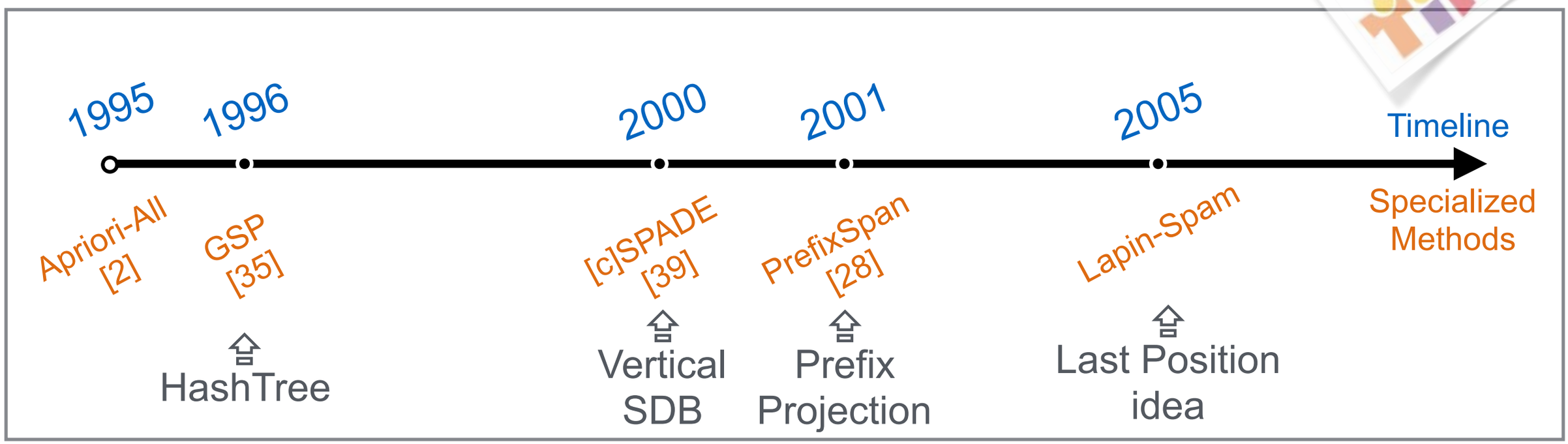


Sequence Database (SDB)

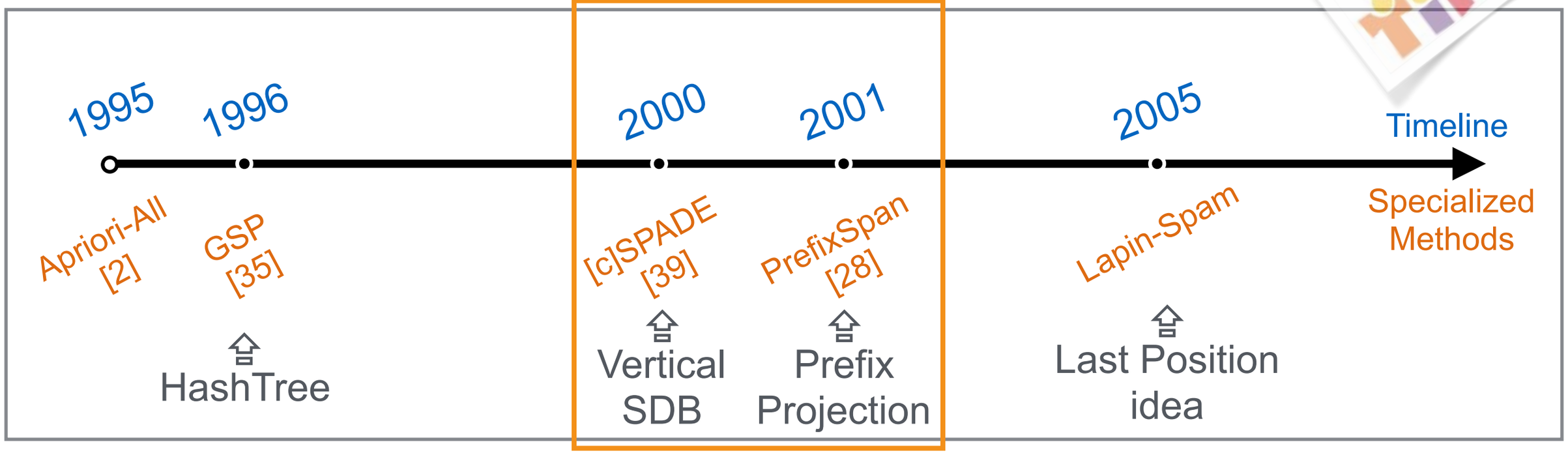
- Sequence : < Milk Coffee Sugar Coffee Sugar>
- Sub-sequence : <Coffee Sugar>
- Support (<Coffee Sugar>) = 3

Problem : Find all subsequences with support ≥ Given Threshold

Related Work



Related Work



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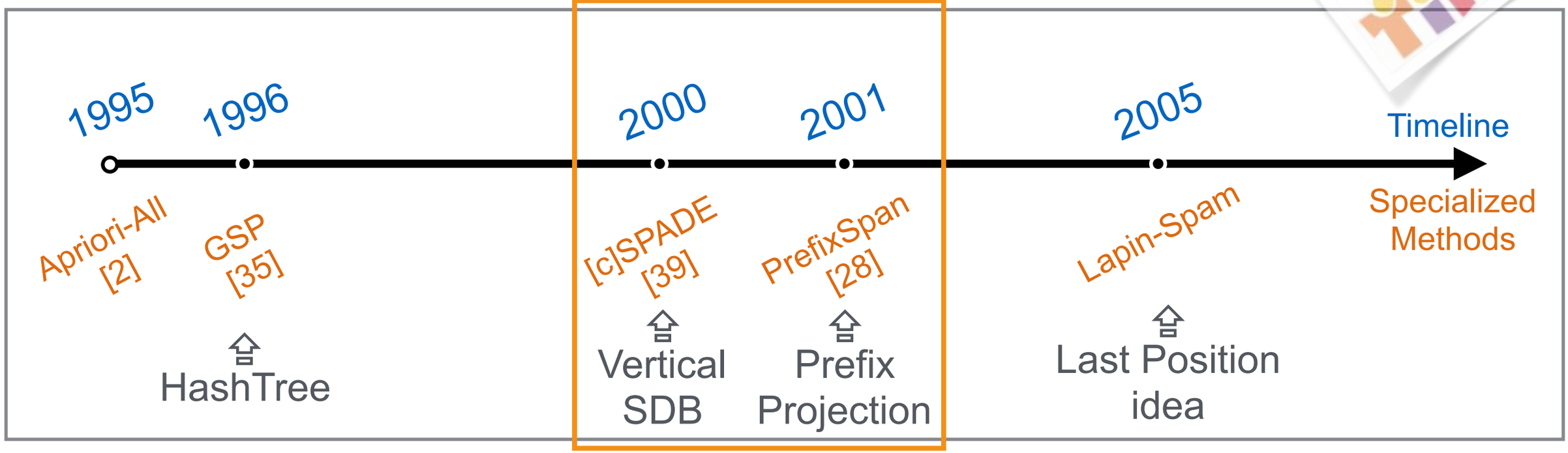

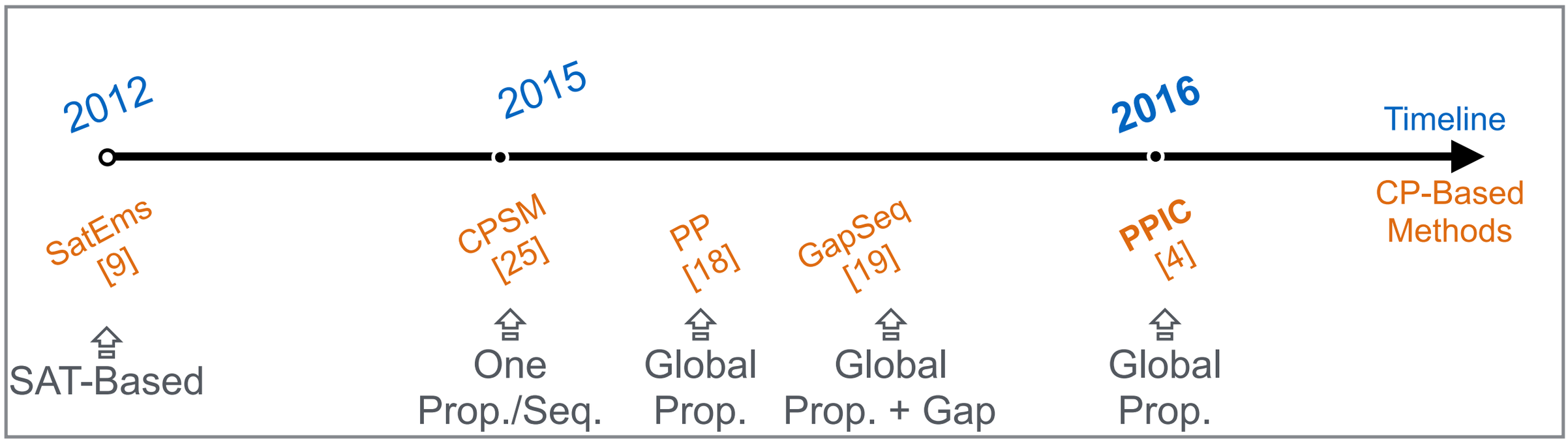

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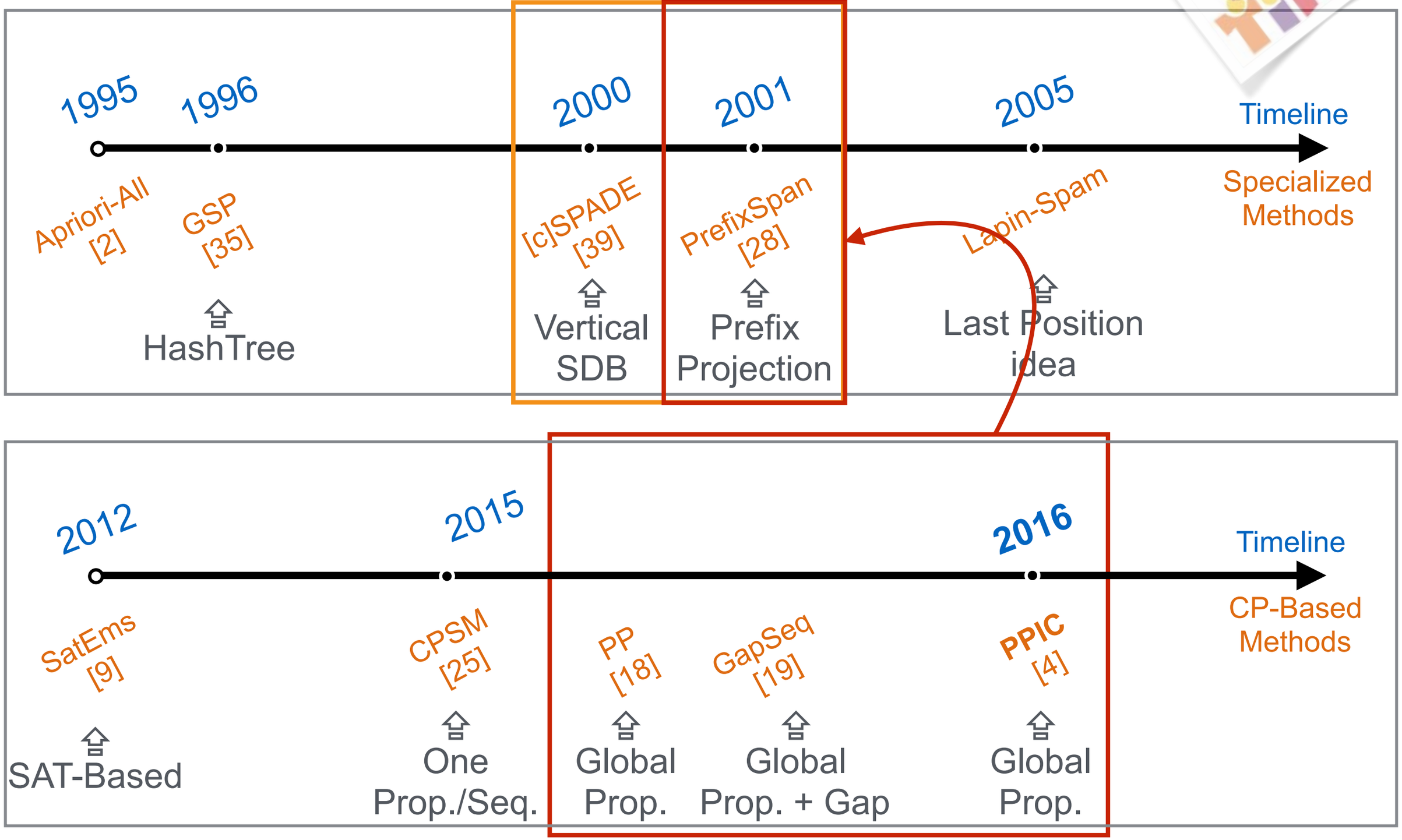

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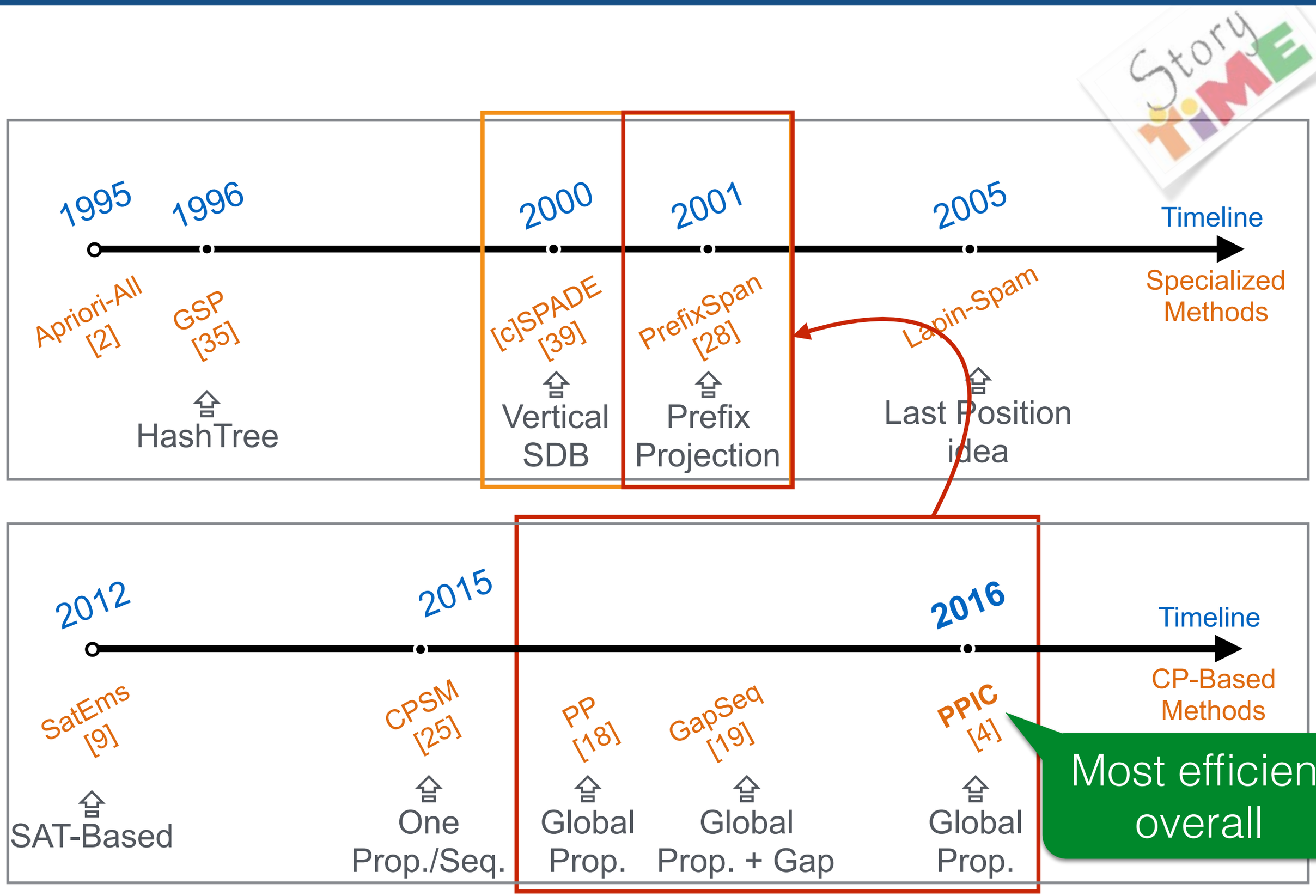

PPICT : CONTRIBUTION

Goal: Capture the most common time-related constraints: namely timed events, minimum/maximum gap and span

☑ *Adapt* trailed-based data structure to efficiently capture **all** valid embeddings (previously only smallest needed)

☑ Algorithmic improvements to avoid scanning overlapping time windows, and to efficiently compute the frequency of symbols

☑ Can be combined with many other contraints: Regular/ Grammar, Gcc, Among, …

CP : Filtering + DFSearch

Vi	P_1	P_2	P_3	P_4	P_5
Di	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg

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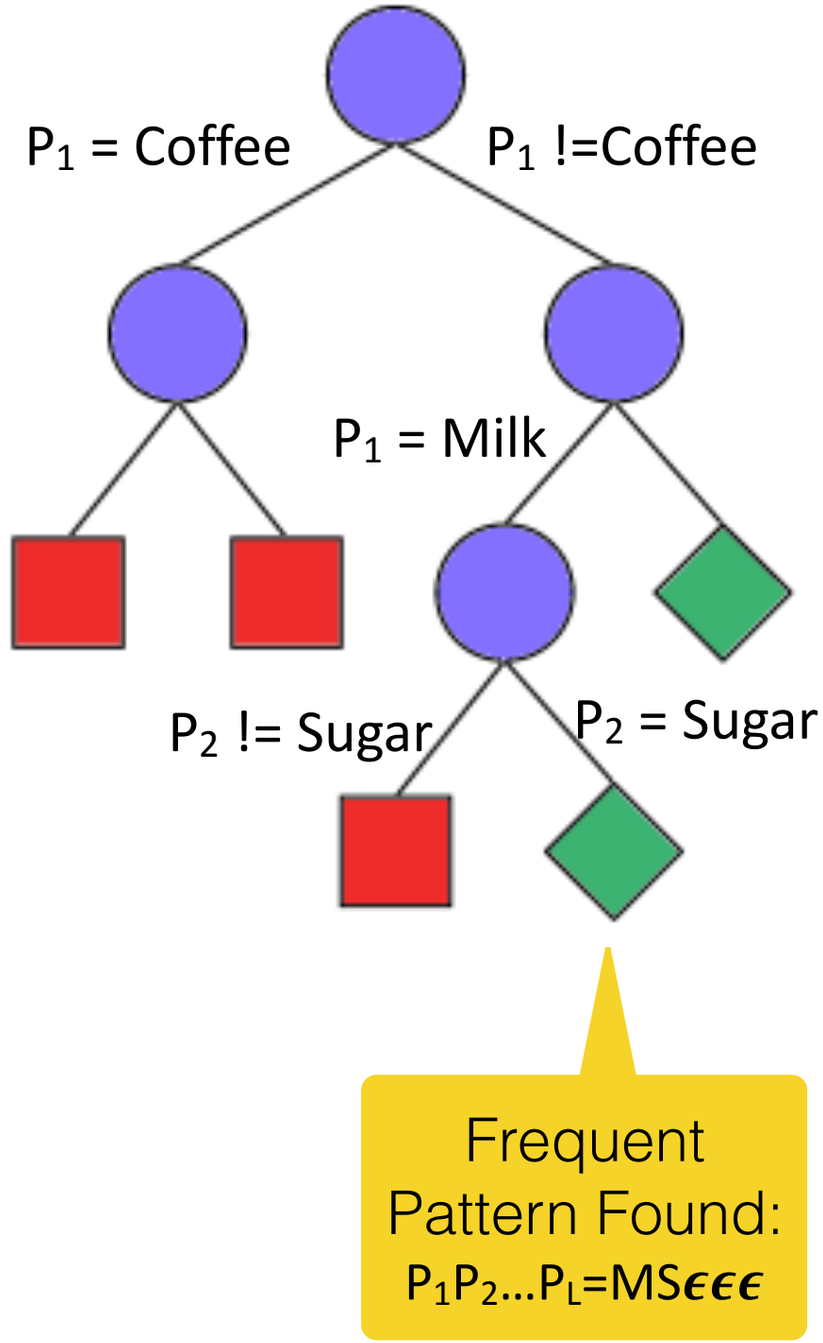

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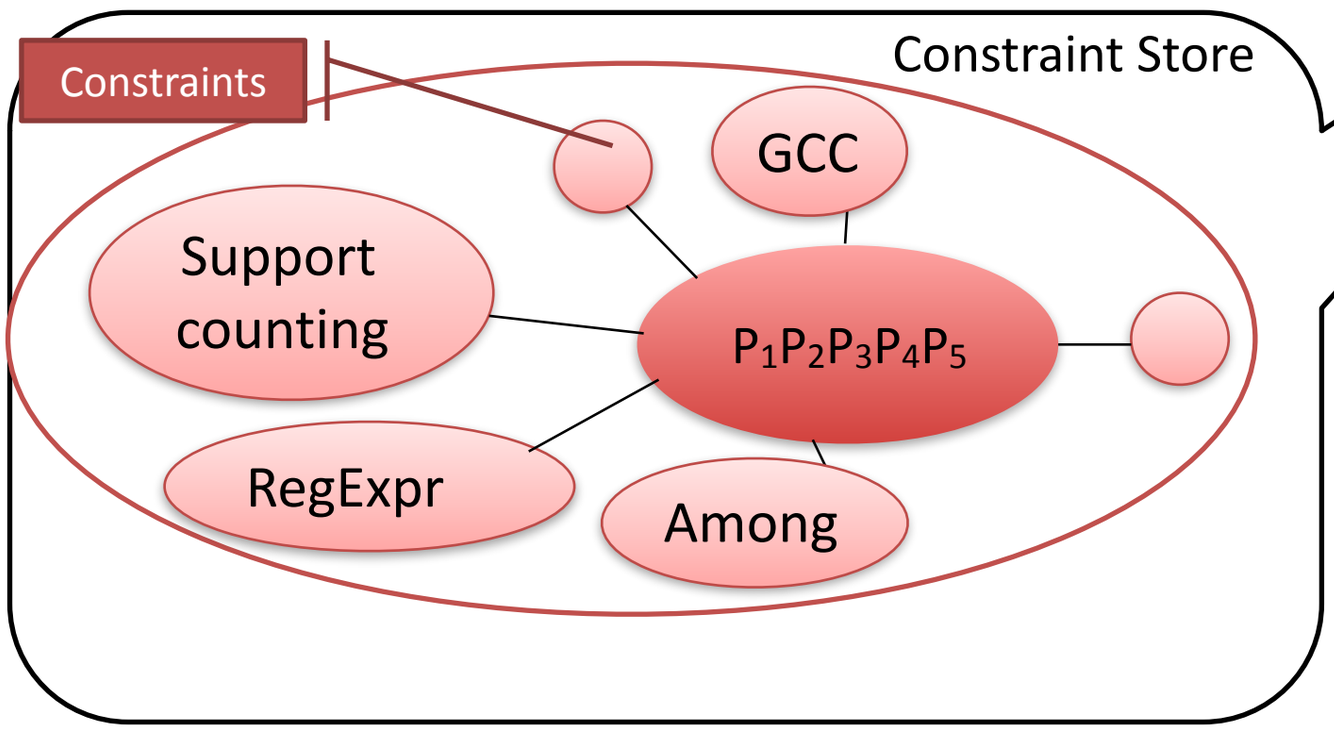

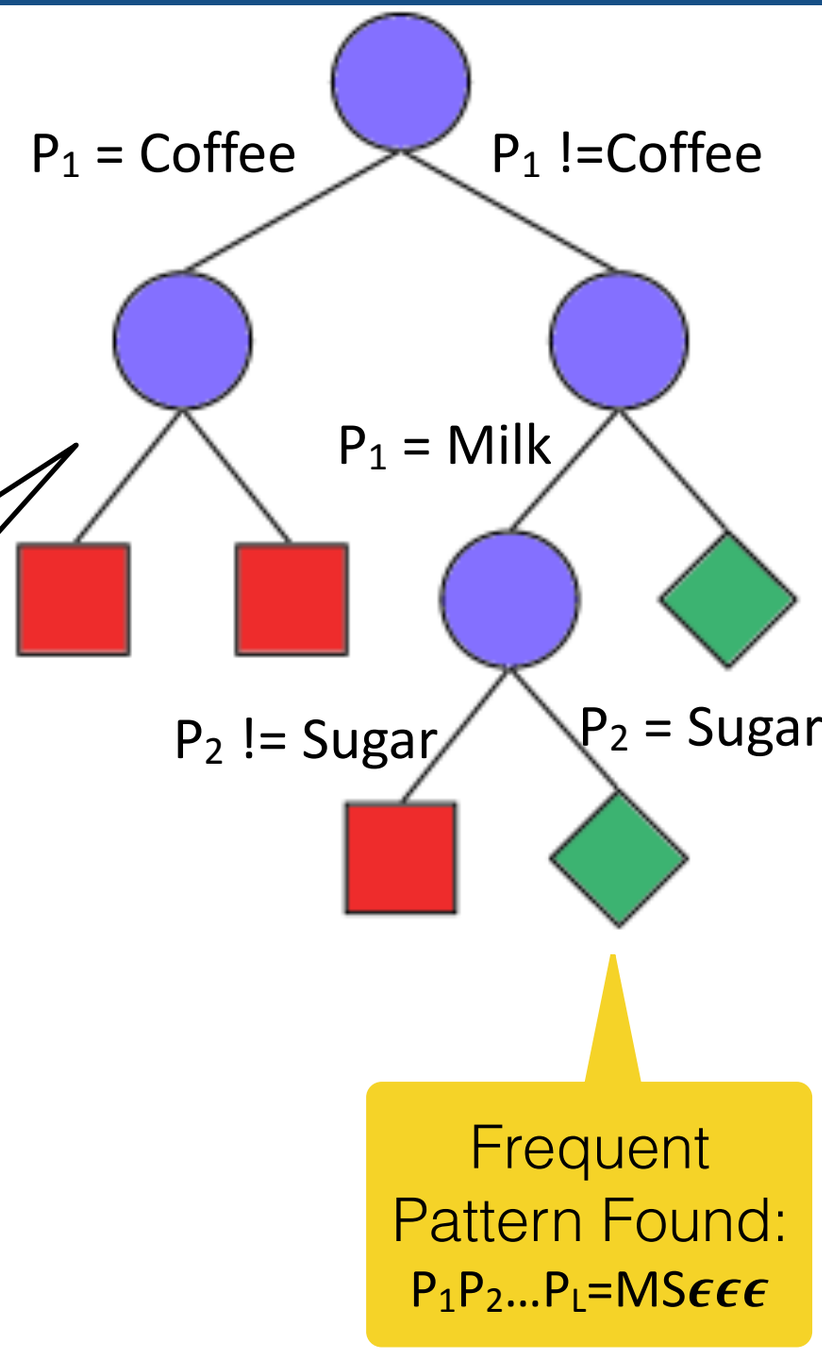

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	Sugar	Sugar	Sugar	Sugar	Sugar
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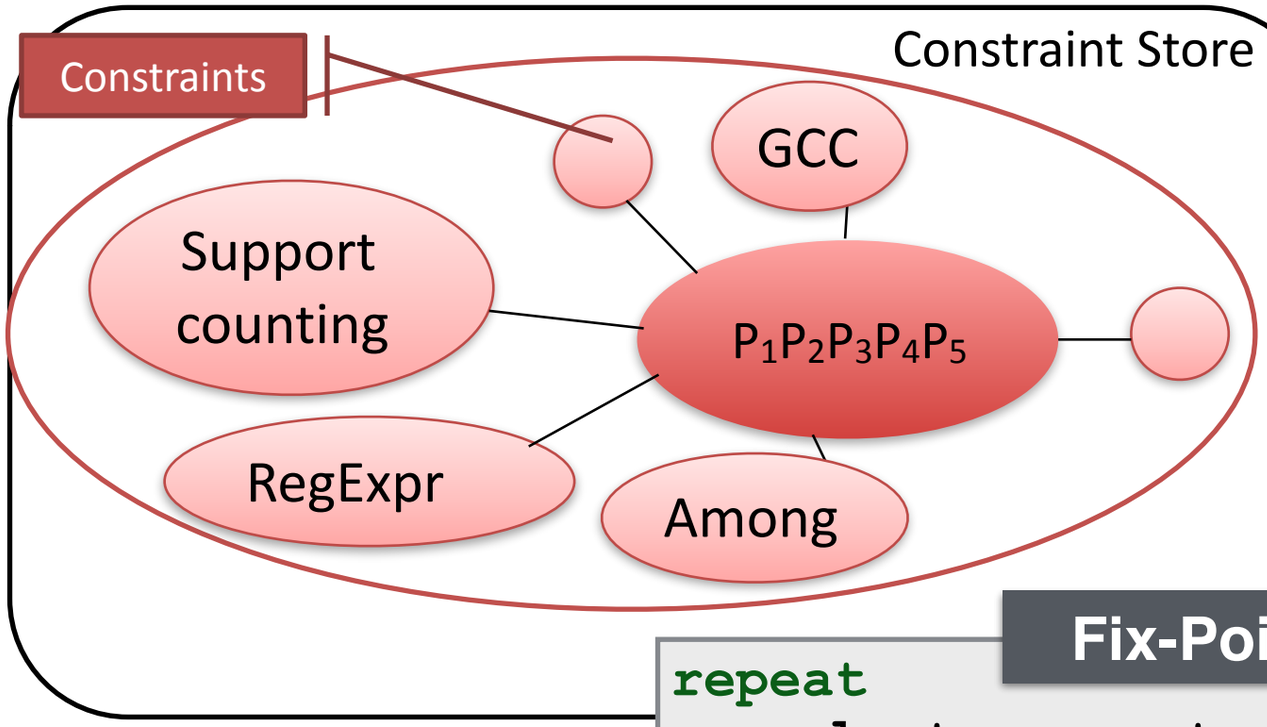

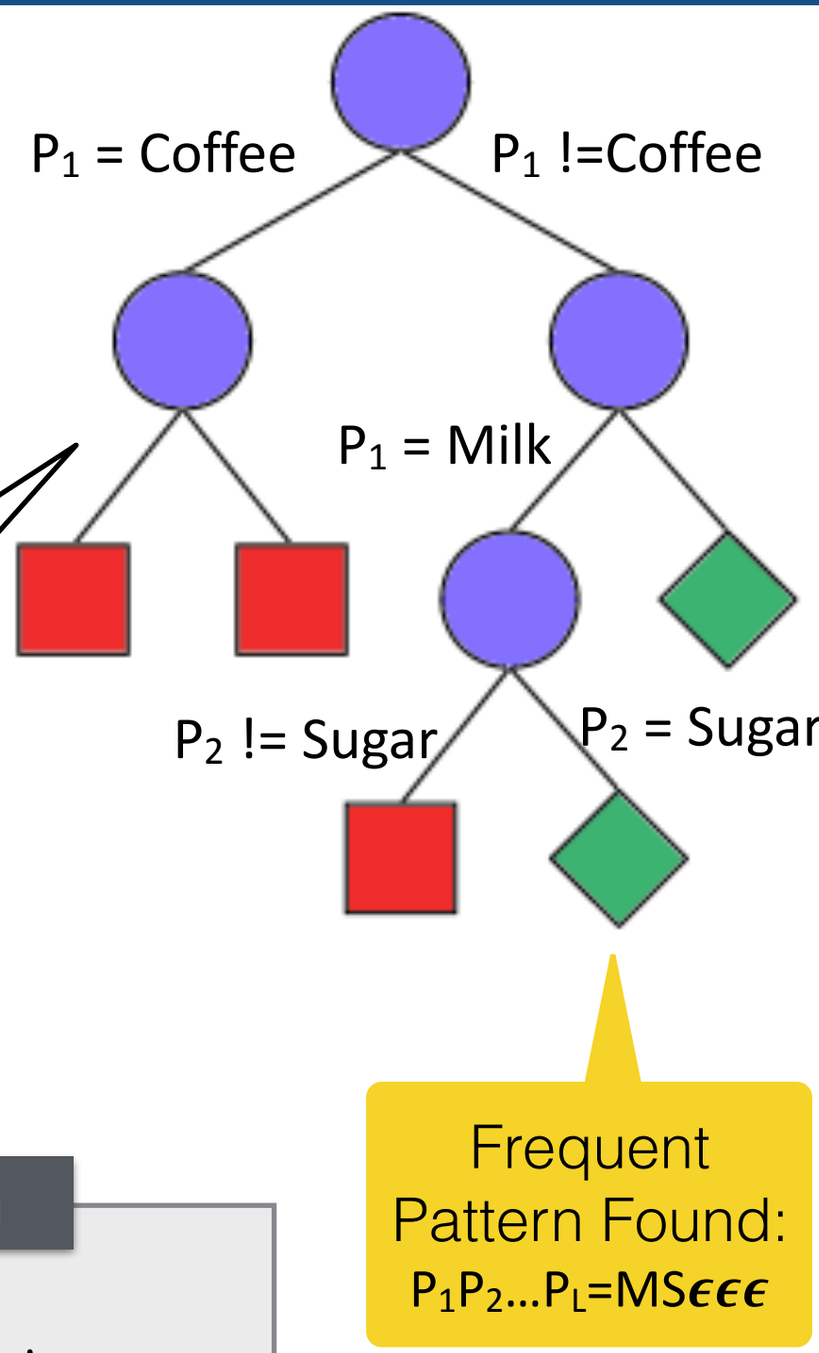

Fix-Point Algorithm

```
repeat
   select a constraint c
   if c is OK wrt the domain store
      apply filtering algorithm of c
     // i.e. remove impossible values
   else
     return FAIL
until domain store did not change
```

CP : Filtering + DFSearch

Vi	P_1	P_2	P_3	P_4	P_5
Di	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk	Milk
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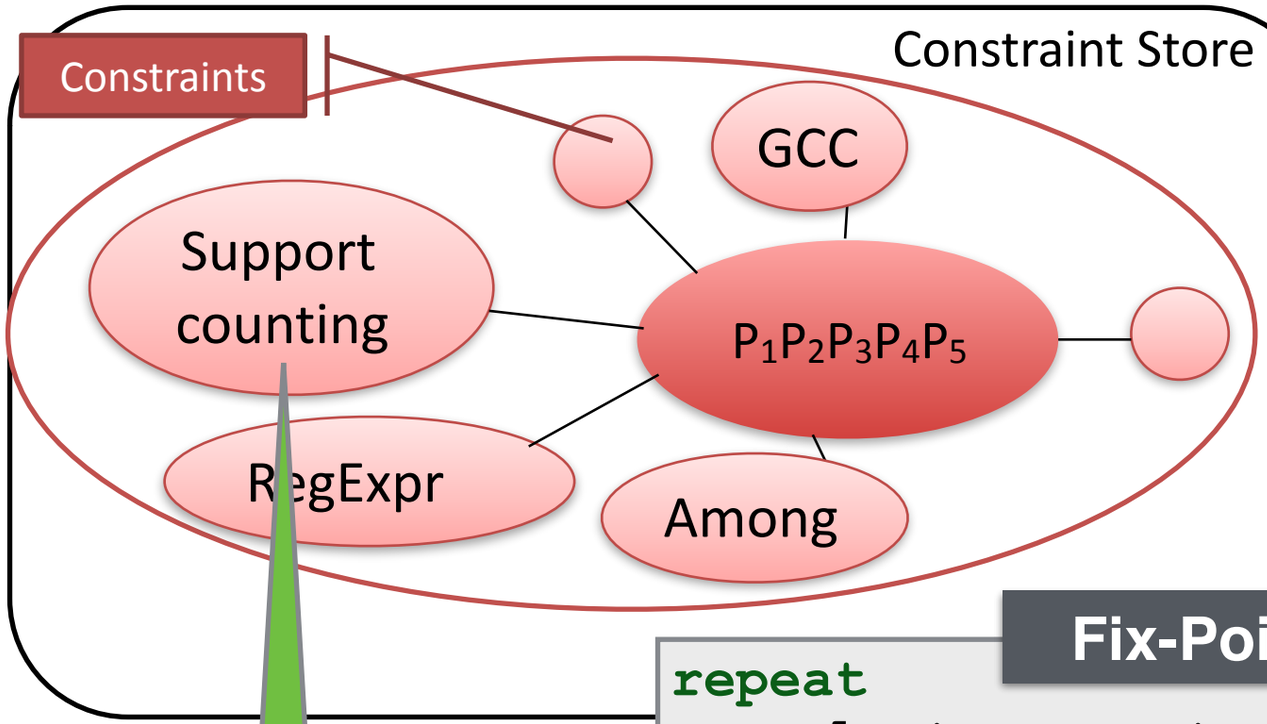

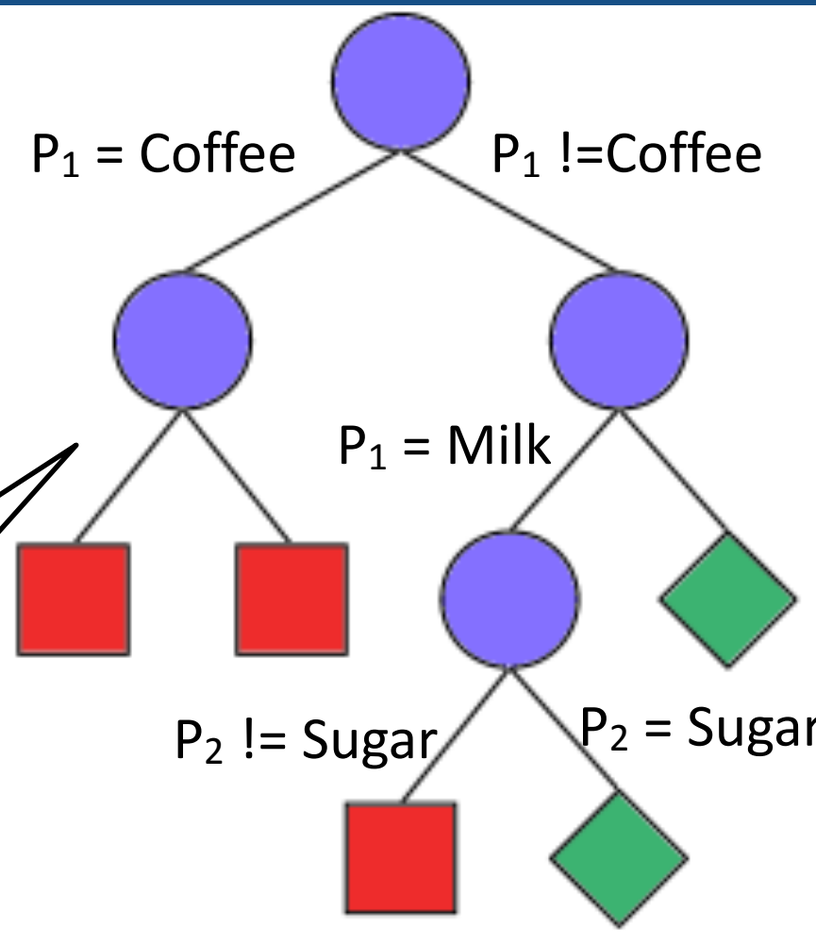

Frequent Pattern Found: $P_1P_2...P_L$=MS$\epsilon\epsilon\epsilon$

This is main bottleneck

Fix-Point Algorithm

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repeat
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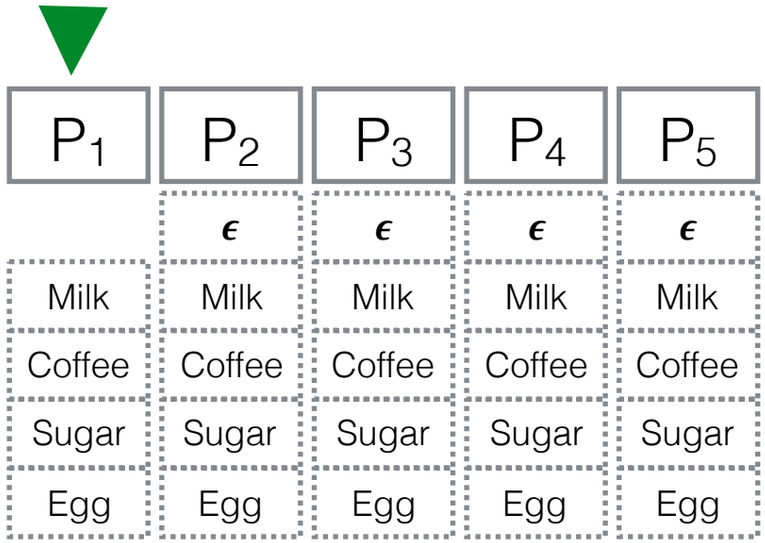
      apply filtering algorithm of c
      // i.e. remove impossible values
   else
     return FAIL
until domain store did not change
```

MinSup=3
(75%)

	0	1	2	3	4
1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
4	C	S	E		

MinSup=3
(75%)

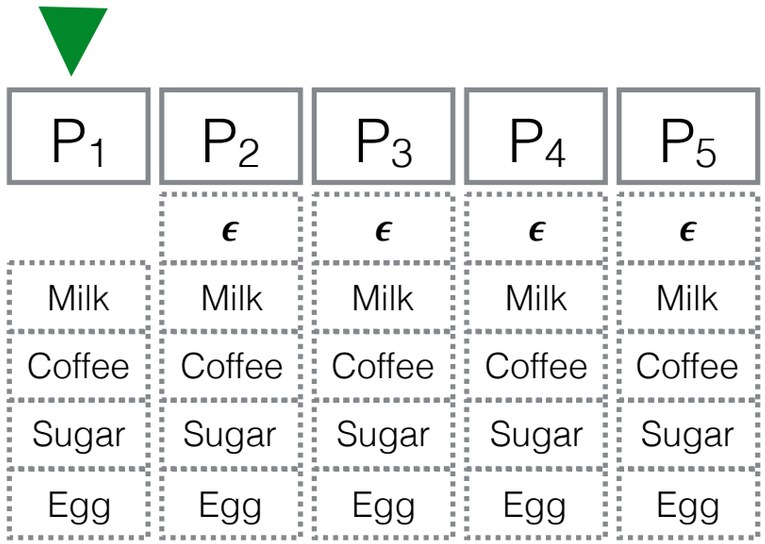
	0	1	2	3	4
1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
4	C	S	E		



MinSup=3
(75%)

	0	1	2	3	4
1	M	C	S	C	S
2	C	**M**	C	S	
3	**M**	C			
4	C	S	E		

Supports
M :



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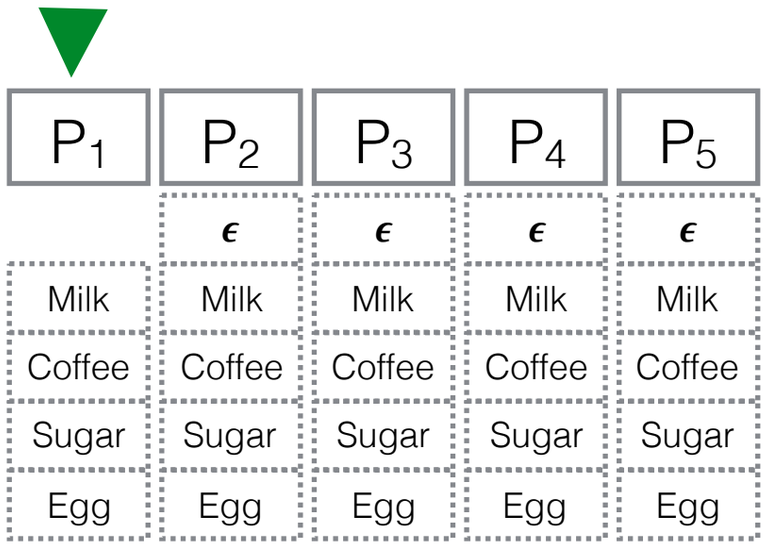
	0	1	2	3	4
1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
4	C	S	E		

Supports
M : 3
C : 4
S : 3
E : 1



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1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
4	C	S	E		

Supports

M : 3

C : 4

S : 3

~~E : 1~~

P_1	P_2	P_3	P_4	P_5
	ϵ	ϵ	ϵ	ϵ
Milk	Milk	Milk	Milk	Milk
Coffee	Coffee	Coffee	Coffee	Coffee
Sugar	Sugar	Sugar	Sugar	Sugar
Egg	Egg	Egg	Egg	Egg

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1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
4	C	S	E		

Supports

M : 3

C : 4

S : 3

~~E : 1~~

P_1	P_2	P_3	P_4	P_5
	ϵ	ϵ	ϵ	ϵ
Milk	Milk	Milk	Milk	Milk
Coffee	Coffee	Coffee	Coffee	Coffee
Sugar	Sugar	Sugar	Sugar	Sugar
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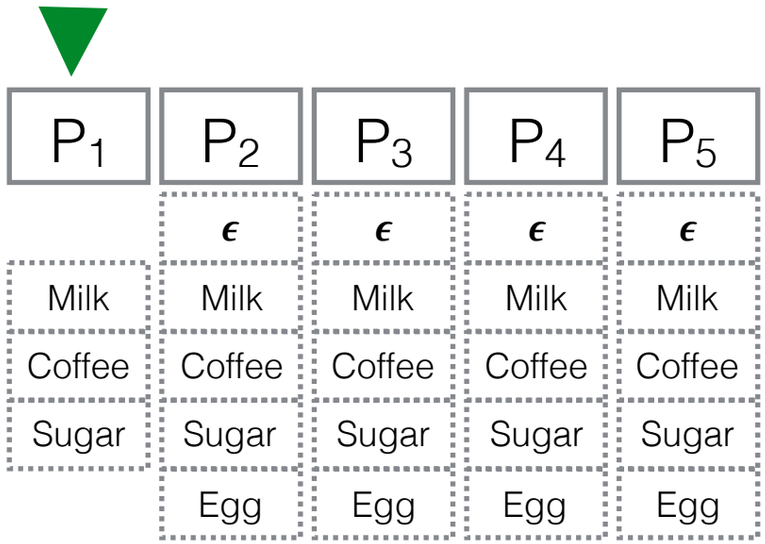
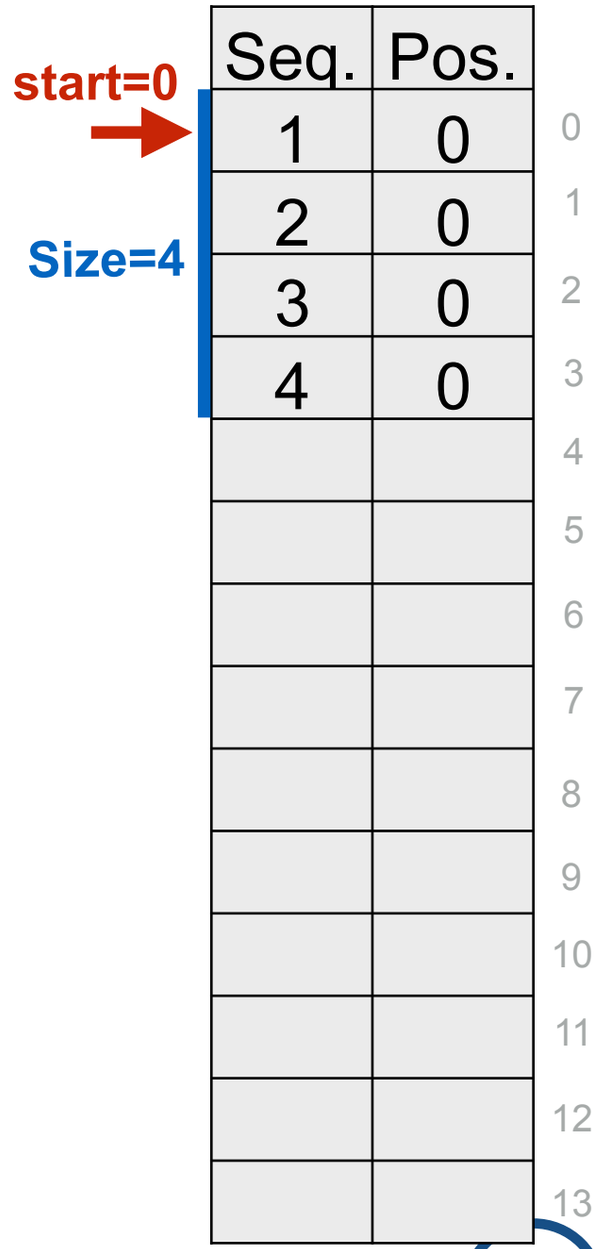
3	M	C			
4	C	S	E		

Supports
M : 3
C : 4
S : 3
~~E : 1~~





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	0	1	2	3	4
1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
4	C	S	E		

M

Supports
M : 3
C : 4
S : 3
~~E : 1~~

P_1	P_2	P_3	P_4	P_5
	ϵ	ϵ	ϵ	ϵ
Milk	Milk	Milk	Milk	Milk
Coffee	Coffee	Coffee	Coffee	Coffee
Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg

start=0

Size=4

	Seq.	Pos.
0	1	0
1	2	0
2	3	0
3	4	0
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		

MinSup=3
(75%)

	0	1	2	3	4
1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
4	C	S	E		

M

Supports
M : 3
C : 4
S : 3
~~E : 1~~

P_1	P_2	P_3	P_4	P_5
	ϵ	ϵ	ϵ	ϵ
Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg

start=0

Size=4

	Seq.	Pos.
0	1	0
1	2	0
2	3	0
3	4	0
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		

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	0	1	2	3	4
1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
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M

	0	1	2	3	4
1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
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Supports
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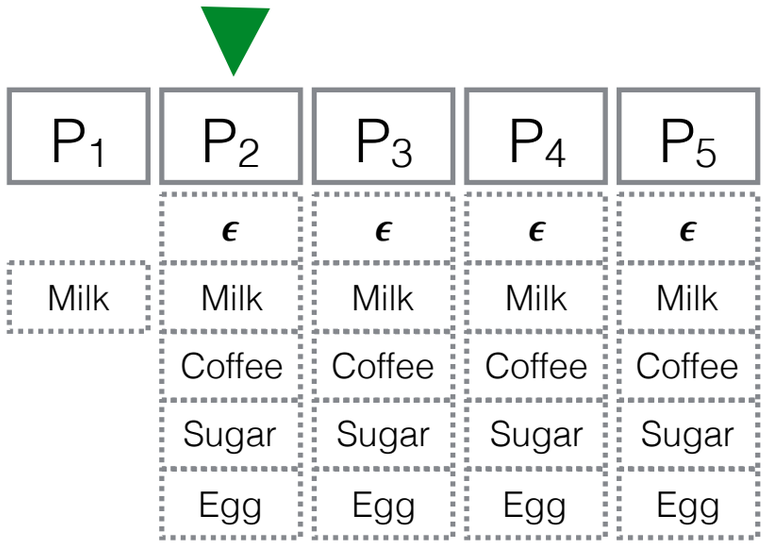

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Size=4

Seq.	Pos.
1	0
2	0
3	0
4	0

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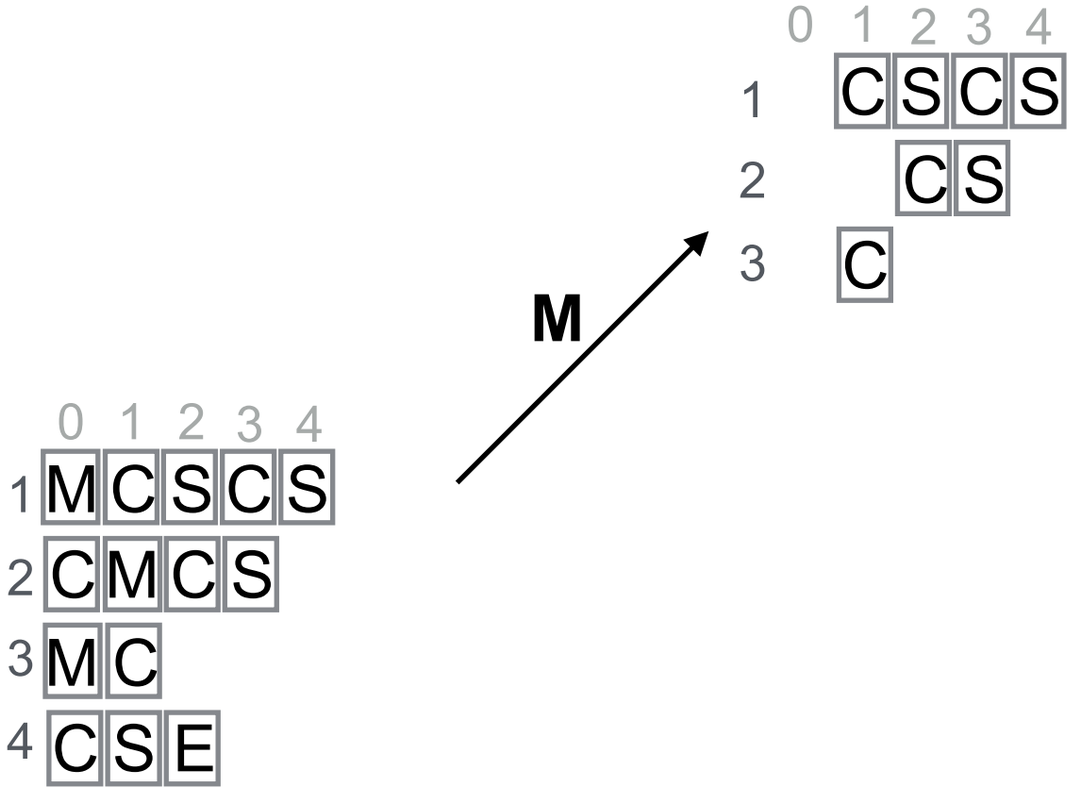

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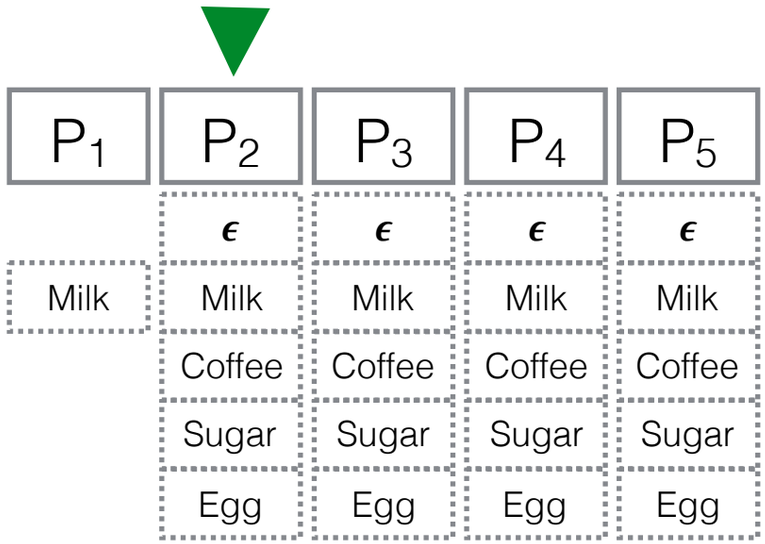

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Seq.	Pos.
1	0
2	0
3	0
4	0

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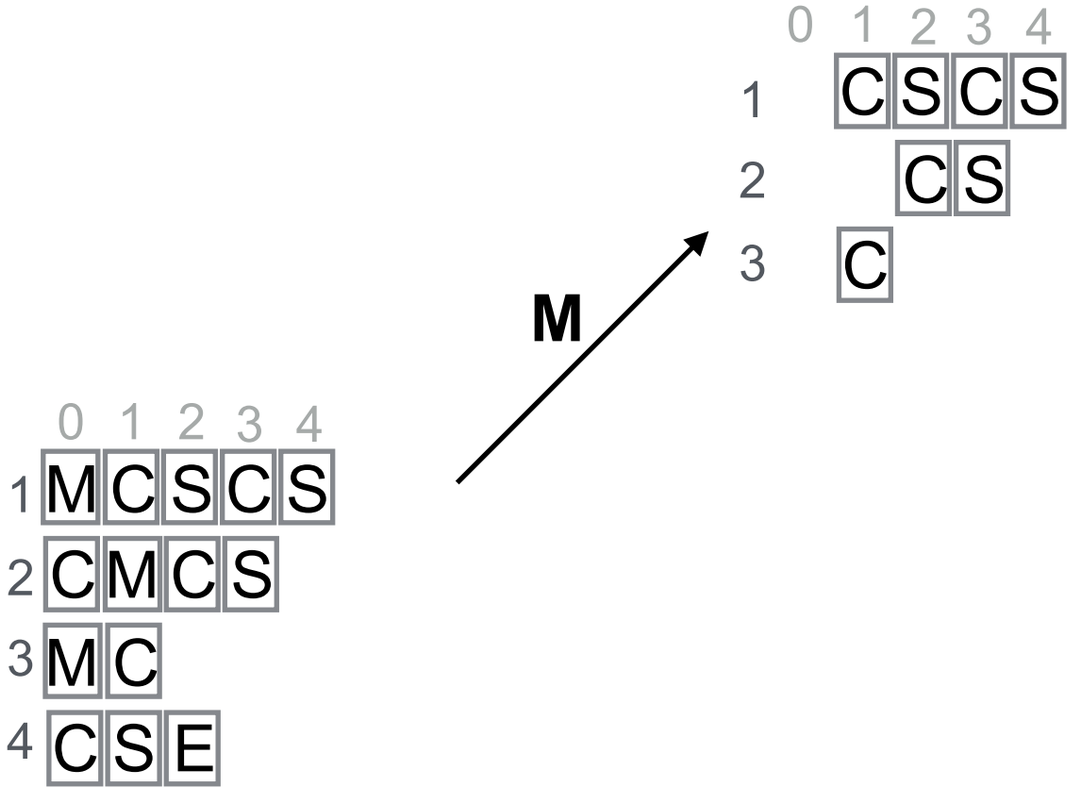

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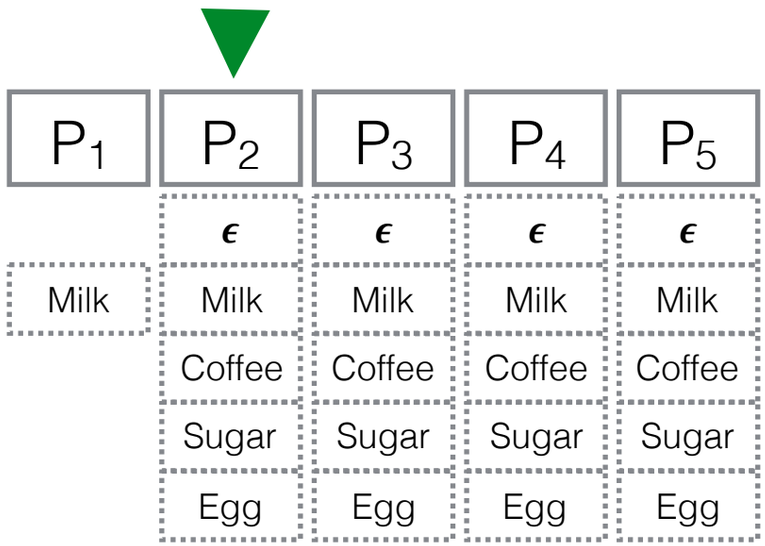

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Size=4

Seq.	Pos.
1	0
2	0
3	0
4	0
1	1
2	2
3	1

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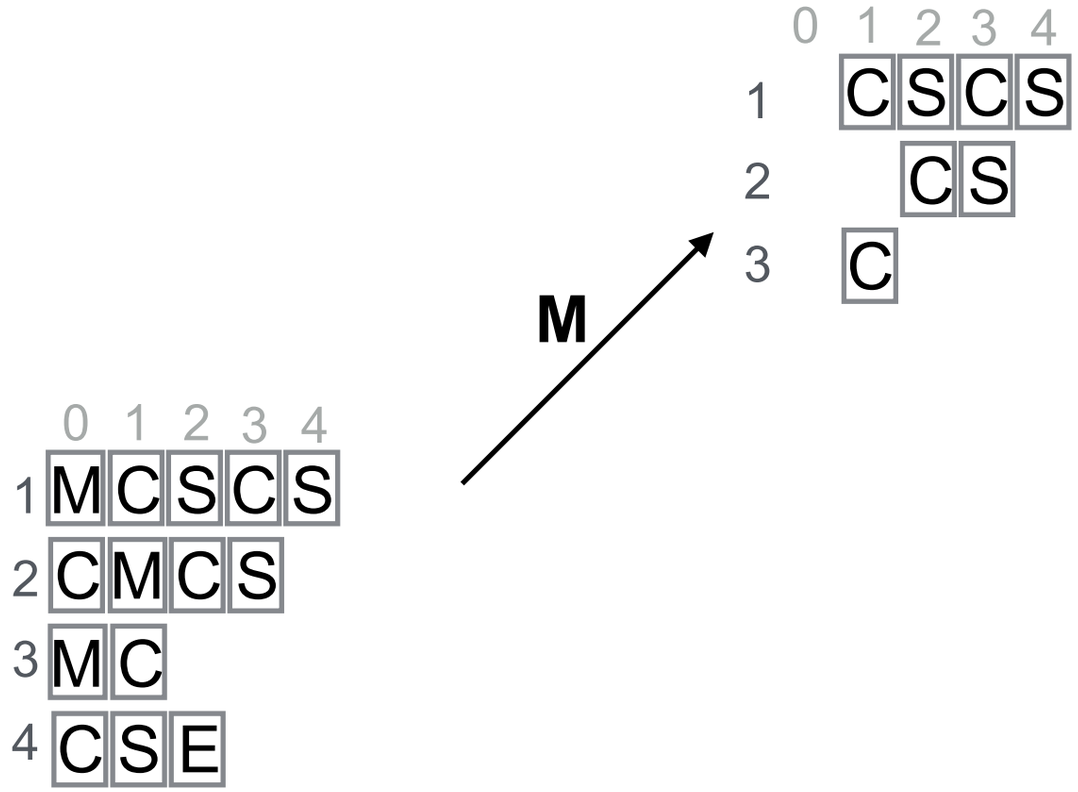

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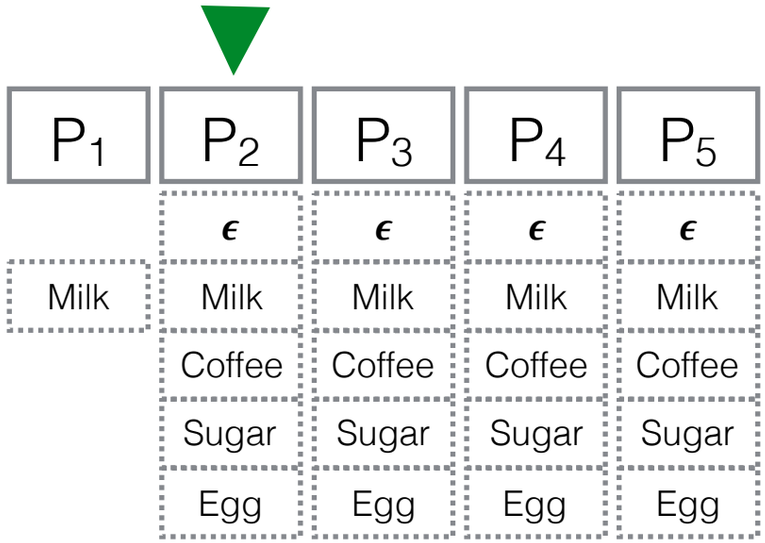


start=4

Size=3

Seq.	Pos.
1	0
2	0
3	0
4	0
1	1
2	2
3	1

MinSup=3
(75%)

	0	1	2	3	4
1	C	S	C	S	
2		C	S		
3	C				

M

	0	1	2	3	4
1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
4	C	S	E		

Supports
M : 3
C : 4
S : 3
~~E : 1~~

P_1	P_2	P_3	P_4	P_5
	ϵ	ϵ	ϵ	ϵ
Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg

Top of the sub-stack

start=0, size=4

TrailStack

Seq.	Pos.
1	0
2	0
3	0
4	0
1	1
2	2
3	1

start=4

Size=3

MinSup=3
(75%)

Supports
M : 0
C : 3
S : 2
~~E : 1~~

	0	1	2	3	4
1	C	S	C	S	
2		C	S		
3	C				

M

	0	1	2	3	4
1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
4	C	S	E		

Supports
M : 3
C : 4
S : 3
~~E : 1~~

P_1	P_2	P_3	P_4	P_5
	ϵ	ϵ	ϵ	ϵ
Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg

Top of the sub-stack

start=0, size=4

TrailStack

start=4

Size=3

Seq.	Pos.
1	0
2	0
3	0
4	0
1	1
2	2
3	1

MinSup=3
(75%)

Supports
~~M : 0~~
C : 3
~~S : 2~~
~~E : 1~~

	0	1	2	3	4
1		C	S	C	S
2			C	S	
3		C			

M

	0	1	2	3	4
1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
4	C	S	E		

Supports
M : 3
C : 4
S : 3
~~E : 1~~

P_1	P_2	P_3	P_4	P_5
	ϵ	ϵ	ϵ	ϵ
Milk		Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee
		Sugar	Sugar	Sugar
		Egg	Egg	Egg

Top of the sub-stack

start=0, size=4

TrailStack

Seq.	Pos.
1	0
2	0
3	0
4	0
1	1
2	2
3	1

start=4

Size=3

MinSup=3
(75%)

Supports
~~M : 0~~
C : 3
~~S : 2~~
~~E : 1~~

	0	1	2	3	4
1	C	S	C	S	
2		C	S		
3	C				

C

Supports
~~M : 0~~
C : 1
~~S : 2~~
~~E : 1~~

	0	1	2	3	4
1	C	S	C	S	
2		C	S		
3	C				

M

	0	1	2	3	4
1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
4	C	S	E		

Supports
M : 3
C : 4
S : 3
~~E : 1~~

P_1	P_2	P_3	P_4	P_5
		ϵ	ϵ	ϵ
Milk		Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee
		Sugar	Sugar	Sugar
		Egg	Egg	Egg

Top of the sub-stack

start=0, size=4

TrailStack

start=4

Size=3

Seq.	Pos.
1	0
2	0
3	0
4	0
1	1
2	2
3	1

MinSup=3
(75%)

Supports
~~M : 0~~
C : 3
~~S : 2~~
~~E : 1~~

	0	1	2	3	4
1	C	S	C	S	
2		C	S		
3	C				

C

Supports
~~M : 0~~
~~C : 1~~
~~S : 2~~
~~E : 1~~

	0	1	2	3	4
1	S	C	S		
2		S			
3					

M

	0	1	2	3	4
1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
4	C	S	E		

Supports
M : 3
C : 4
S : 3
~~E : 1~~

P_1	P_2	P_3	P_4	P_5
		ϵ	ϵ	ϵ
Milk		Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee
		Sugar	Sugar	Sugar
		Egg	Egg	Egg

Top of the sub-stack

start=0, size=4

TrailStack

start=4

Size=3

Seq.	Pos.
1	0
2	0
3	0
4	0
1	1
2	2
3	1

MinSup=3
(75%)

Supports
~~M : 0~~
C : 3
~~S : 2~~
~~E : 1~~

	0	1	2	3	4
1	C	S	C	S	
2		C	S		
3	C				

C

Supports
~~M : 0~~
~~C : 1~~
~~S : 2~~
~~E : 1~~

	0	1	2	3	4
1	S	C	S		
2		S			
3					

M

	0	1	2	3	4
1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
4	C	S	E		

Supports
M : 3
C : 4
S : 3
~~E : 1~~

P_1	P_2	P_3	P_4	P_5
		ϵ	ϵ	ϵ
Milk		Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee
		Sugar	Sugar	Sugar
		Egg	Egg	Egg

Top of the sub-stack

start=4

start=0, size=4

TrailStack

start=7

Size=3

Seq.	Pos.
1	0
2	0
3	0
4	0
1	1
2	2
3	1
1	2
2	3
3	2

MinSup=3
(75%)

Supports
~~M : 0~~
C : 3
~~S : 2~~
~~E : 1~~

	0	1	2	3	4
1	C	S	C	S	
2		C	S		
3	C				

C

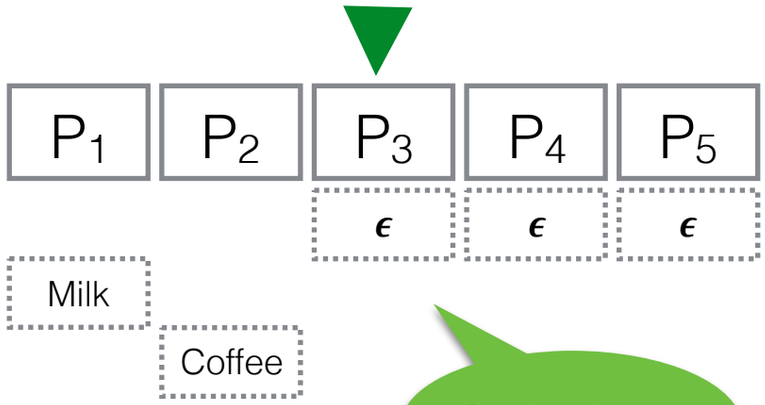
Supports
~~M : 0~~
~~C : 1~~
~~S : 2~~
~~E : 1~~

	0	1	2	3	4
1	S	C	S		
2		S			
3					

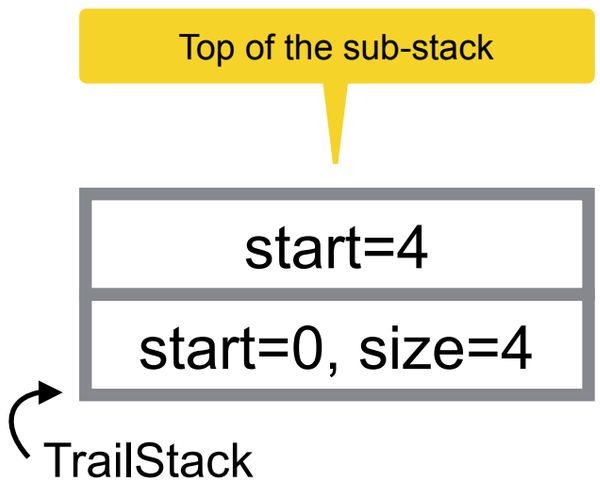
M

	0	1	2	3	4
1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
4	C	S	E		

Supports
M : 3
C : 4
S : 3
~~E : 1~~



Solution!



Seq.	Pos.
1	0
2	0
3	0
4	0
1	1
2	2
3	1
1	2
2	3
3	2

start=7

Size=3

MinSup=3
(75%)

Supports
~~M : 0~~
C : 3
~~S : 2~~
~~E : 1~~

	0	1	2	3	4
1	C	S	C	S	
2		C	S		
3	C				

C

Supports
~~M : 0~~
~~C : 1~~
~~S : 2~~
~~E : 1~~

	0	1	2	3	4
1	S	C	S		
2		S			
3					

Backtrack

M

	0	1	2	3	4
1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
4	C	S	E		

Supports
M : 3
C : 4
S : 3
~~E : 1~~

P_1	P_2	P_3	P_4	P_5
	ϵ	ϵ	ϵ	ϵ
Milk		Milk	Milk	Milk
		Coffee	Coffee	Coffee
		Sugar	Sugar	Sugar
		Egg	Egg	Egg

Top of the sub-stack

start=0, size=4

TrailStack

start=4

Size=3

Seq.	Pos.
1	0
2	0
3	0
4	0
1	1
2	2
3	1
1	2
2	3
3	2

MinSup=3
(75%)

Supports
~~M : 0~~
C : 3
~~S : 2~~
~~E : 1~~

	0	1	2	3	4
1	C	S	C	S	
2		C	S		
3	C				

C

Supports
~~M : 0~~
~~C : 1~~
~~S : 2~~
~~E : 1~~

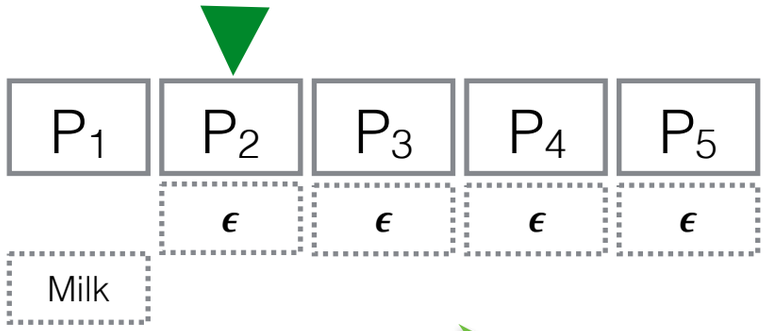
	0	1	2	3	4
1	S	C	S		
2		S			
3					

Backtrack

M

	0	1	2	3	4
1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
4	C	S	E		

Supports
M : 3
C : 4
S : 3
~~E : 1~~



Top of the sub-stack

Solution!

start=0, size=4

TrailStack

start=4

Size=3

Seq.	Pos.
1	0
2	0
3	0
4	0
1	1
2	2
3	1
1	2
2	3
3	2

MinSup=3
(75%)

Supports
~~M : 0~~
C : 3
~~S : 2~~
~~E : 1~~

	0	1	2	3	4
1	C	S	C	S	
2		C	S		
3	C				

C

Supports
~~M : 0~~
~~C : 1~~
~~S : 2~~
~~E : 1~~

	0	1	2	3	4
1	S	C	S		
2		S			
3					

Backtrack

M

	0	1	2	3	4
1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
4	C	S	E		

Backtrack

Supports
M : 3
C : 4
S : 3
~~E : 1~~

P_1	P_2	P_3	P_4	P_5
	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk
Coffee	Coffee	Coffee	Coffee	Coffee
Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg

Top of the sub-stack

TrailStack

start=0

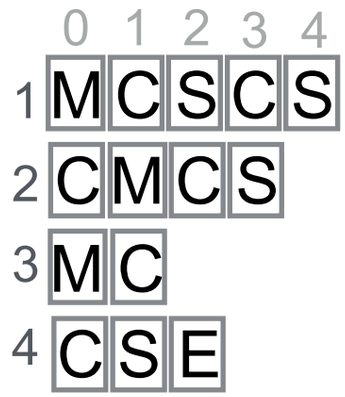
Size=4

Seq.	Pos.
1	0
2	0
3	0
4	0
1	1
2	2
3	1
1	2
2	3
3	2

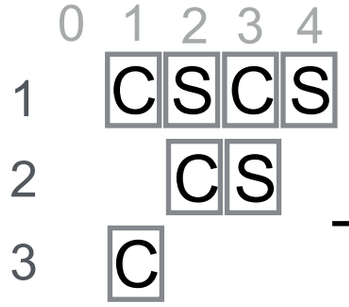
MinSup=3
(75%)

Supports
~~M : 0~~
C : 3
~~S : 2~~
~~E : 1~~

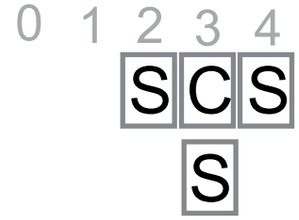
Supports
~~M : 0~~
~~C : 1~~
~~S : 2~~
~~E : 1~~



M



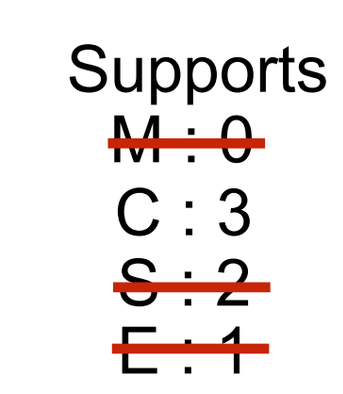
C



Backtrack

Backtrack

C



start=0

Size=4

Seq.	Pos.
1	0
2	0
3	0
4	0
1	1
2	2
3	1
1	2
2	3
3	2

Supports
M : 3
C : 4
S : 3
~~E : 1~~

P_1	P_2	P_3	P_4	P_5
	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk
Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg

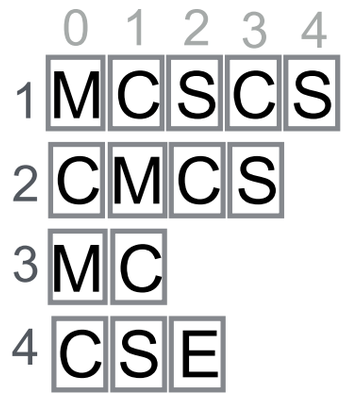
Top of the sub-stack

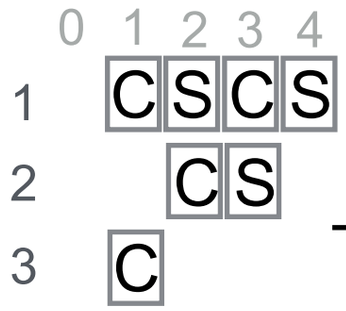
TrailStack

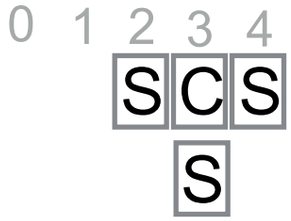
MinSup=3
(75%)

Supports
~~M : 0~~
C : 3
~~S : 2~~
~~E : 1~~

Supports
~~M : 0~~
~~C : 1~~
~~S : 2~~
~~E : 1~~



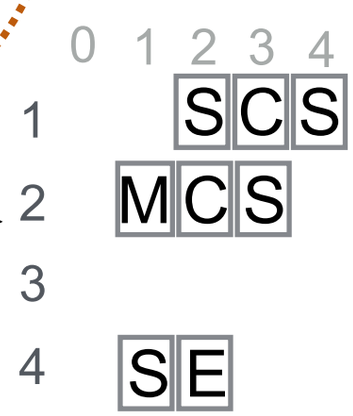




M

C

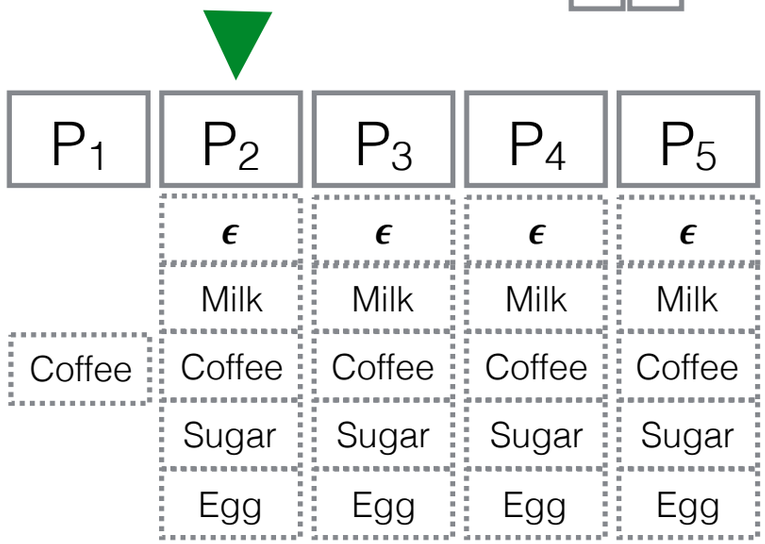
Backtrack



Backtrack

C

Supports
M : 3
C : 4
S : 3
~~E : 1~~



Top of the sub-stack

TrailStack

start=0

Size=4

Seq.	Pos.
1	0
2	0
3	0
4	0
1	1
2	2
3	1
1	2
2	3
3	2

MinSup=3
(75%)

Supports
~~M : 0~~
C : 3
~~S : 2~~
~~E : 1~~

Supports
~~M : 0~~
~~C : 1~~
~~S : 2~~
~~E : 1~~

	0	1	2	3	4
1	C	S	C	S	
2		C	S		
3	C				

C

	0	1	2	3	4
1		S	C	S	
2	M	C	S		
3					
4	S	E			

C

	0	1	2	3	4
1	S	C	S		
2		S			

M

	0	1	2	3	4
1	M	C	S	C	S
2	C	M	C	S	
3	M	C			
4	C	S	E		

Backtrack

Backtrack

Supports
M : 3
C : 4
S : 3
~~E : 1~~

P_1	P_2	P_3	P_4	P_5
	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk
Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg

Top of the sub-stack

start=0

TrailStack

start=4

Size=4

Seq.	Pos.
1	0
2	0
3	0
4	0
1	2
2	1
3	2
4	1
2	3
3	2

TIME DATABASE CHALLENGES

Gap[M,N] = Minimum(M) and Maximum(N) gap time restriction

Span[Y,W] = Minimum(Y) and Maximum(W) span time restriction

span=8

gap=3

gap=5

Client1	(2, Milk)	(5, Coffee)	(6, Egg)	(10, Sugar)	(11, Coffee)		
Client2	(1, Coffee)	(2, Milk)	(9, Milk)	(12, Egg)	(15, Sugar)	(18, Milk)	(24, Coffee)
Client3	(2, Milk)	(4, Coffee)	(6, Egg)	(8, Egg)	(10, Coffee)	(12, Wine)	(14, Sugar)
Client4	(2, Milk)	(5, Sugar)	(6, Sugar)	(10, Coffee)			

Sequence Database (SDB)

TIME DATABASE CHALLENGES

Gap[M,N] = Minimum(M) and Maximum(N) gap time restriction

Span[Y,W] = Minimum(Y) and Maximum(W) span time restriction

span=8

gap=3 gap=5

Client1	(2, Milk)	(5, Coffee)	(6, Egg)	(10, Sugar)	(11, Coffee)		
Client2	(1, Coffee)	(2, Milk)	(9, Milk)	(12, Egg)	(15, Sugar)	(18, Milk)	(24, Coffee)
Client3	(2, Milk)	(4, Coffee)	(6, Egg)	(8, Egg)	(10, Coffee)	(12, Wine)	(14, Sugar)
Client4	(2, Milk)	(5, Sugar)	(6, Sugar)	(10, Coffee)			

Sequence Database (SDB)

Client1

- gap[3,7] (<(2, Milk)(6, Egg)(10, Sugar)>)
- gap[3,7] (<(2, Milk)(10, Sugar)>)

TIME DATABASE CHALLENGES

Gap[M,N] = Minimum(M) and Maximum(N) gap time restriction

Span[Y,W] = Minimum(Y) and Maximum(W) span time restriction

span=8

gap=3 gap=5

Client1	(2, Milk)	(5, Coffee)	(6, Egg)	(10, Sugar)	(11, Coffee)		
Client2	(1, Coffee)	(2, Milk)	(9, Milk)	(12, Egg)	(15, Sugar)	(18, Milk)	(24, Coffee)
Client3	(2, Milk)	(4, Coffee)	(6, Egg)	(8, Egg)	(10, Coffee)	(12, Wine)	(14, Sugar)
Client4	(2, Milk)	(5, Sugar)	(6, Sugar)	(10, Coffee)			

Sequence Database (SDB)

Client1

- gap[3,7] (<(2, Milk)(6, Egg)(10, Sugar)>)
- gap[3,7] (<(2, Milk)(10, Sugar)>)

non anti-monotone

TIME DATABASE CHALLENGES

Gap[M,N] = Minimum(M) and Maximum(N) gap time restriction

Span[Y,W] = Minimum(Y) and Maximum(W) span time restriction

span=8

gap=3 gap=5

Client1	(2, Milk)	(5, Coffee)	(6, Egg)	(10, Sugar)	(11, Coffee)		
Client2	(1, Coffee)	(2, Milk)	(9, Milk)	(12, Egg)	(15, Sugar)	(18, Milk)	(24, Coffee)
Client3	(2, Milk)	(4, Coffee)	(6, Egg)	(8, Egg)	(10, Coffee)	(12, Wine)	(14, Sugar)
Client4	(2, Milk)	(5, Sugar)	(6, Sugar)	(10, Coffee)			

Sequence Database (SDB)

Client1
- gap[3,7] (<(2, Milk)(6, Egg)(10, Sugar)>)
- gap[3,7] (<(2, Milk)(10, Sugar)>)

Client2
- gap[3,7] (<(2, Milk)(12, Egg)(15, Sugar)>)
- gap[3,7] (<(9, Milk)(12, Egg)(15, Sugar)>)

non anti-monotone

TIME DATABASE CHALLENGES

Gap[M,N] = Minimum(M) and Maximum(N) gap time restriction

Span[Y,W] = Minimum(Y) and Maximum(W) span time restriction

span=8

gap=3 gap=5

Client1	(2, Milk)	(5, Coffee)	(6, Egg)	(10, Sugar)	(11, Coffee)		
Client2	(1, Coffee)	(2, Milk)	(9, Milk)	(12, Egg)	(15, Sugar)	(18, Milk)	(24, Coffee)
Client3	(2, Milk)	(4, Coffee)	(6, Egg)	(8, Egg)	(10, Coffee)	(12, Wine)	(14, Sugar)
Client4	(2, Milk)	(5, Sugar)	(6, Sugar)	(10, Coffee)			

Sequence Database (SDB)

Client1
- gap[3,7] (<(2, Milk)(6, Egg)(10, Sugar)>)
- gap[3,7] (<(2, Milk)(10, Sugar)>)

non anti-monotone

Client2
- gap[3,7] (<(2, Milk)(12, Egg)(15, Sugar)>)
- gap[3,7] (<(9, Milk)(12, Egg)(15, Sugar)>)

Prefix notion non-applicable

MinSup=3
Gap[3,7] t.u.

	1	2	3	4	5	6	7
1	2:**M**	5:C	6:E	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	2:M	4:C	6:E	8:E	10:C	12:W	14:S
4	1:M	2:S	3:S	4:C			

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di	ϵ	ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg	Egg	Egg
	Wine	Wine	Wine	Wine	Wine	Wine	Wine

seq	size	emb **(start:end)**		
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MinSup=3
Gap[3,7] t.u.

	1	2	3	4	5	6	7
1	2:**M**	5:C	6:E	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	2:M	4:C	6:E	8:E	10:C	12:W	14:S
4	1:M	2:S	3:S	4:C			

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di	ϵ	ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg	Egg	Egg
	Wine	Wine	Wine	Wine	Wine	Wine	Wine

seq	size	emb **(start:end)**		
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MinSup=3
Gap[3,7] t.u.

	1	2	3	4	5	6	7
1	2:**M**	5:C	6:E	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	2:M	4:C	6:E	8:E	10:C	12:W	14:S
4	1:M	2:S	3:S	4:C			

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg	Egg	Egg
	Wine	Wine	Wine	Wine	Wine	Wine	Wine

seq	size	emb **(start:end)**		
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MinSup=3
Gap[3,7] t.u.

	1	2	3	4	5	6	7
1	2:**M**	5:C	6:E	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	2:M	4:C	6:E	8:E	10:C	12:W	14:S
4	1:M	2:S	3:S	4:C			

Supports M :

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg	Egg	Egg
	Wine	Wine	Wine	Wine	Wine	Wine	Wine

seq	size	emb **(start:end)**		
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MinSup=3
Gap[3,7] t.u.

	1	2	3	4	5	6	7
1	2:M	5:C	6:E	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	2:M	4:C	6:E	8:E	10:C	12:W	14:S
4	1:M	2:S	3:S	4:C			

Supports
M : 4
C : 4
S : 4
E : 3
W : 1

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg	Egg	Egg
	Wine	Wine	Wine	Wine	Wine	Wine	Wine

seq	size	emb **(start:end)**		
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MinSup=3
Gap[3,7] t.u.

	1	2	3	4	5	6	7
1	2:M	5:C	6:E	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	2:M	4:C	6:E	8:E	10:C	12:W	14:S
4	1:M	2:S	3:S	4:C			

Supports
M : 4
C : 4
S : 4
E : 3
~~W : 1~~

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg	Egg	Egg
	Wine	Wine	Wine	Wine	Wine	Wine	Wine

seq	size	emb **(start:end)**		
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MinSup=3
Gap[3,7] t.u.

	1	2	3	4	5	6	7
1	2:M	5:C	6:E	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	2:M	4:C	6:E	8:E	10:C	12:W	14:S
4	1:M	2:S	3:S	4:C			

Supports
M : 4
C : 4
S : 4
E : 3
~~W : 1~~

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg	Egg	Egg
	Wine	Wine	Wine	Wine	Wine	Wine	Wine

seq	size	emb **(start:end)**		
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MinSup=3
Gap[3,7] t.u.

	1	2	3	4	5	6	7
1	2:M	5:C	6:E	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	2:M	4:C	6:E	8:E	10:C	12:W	14:S
4	1:M	2:S	3:S	4:C			

Supports
M : 4
C : 4
S : 4
E : 3
~~W : 1~~

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg	Egg	Egg
		Wine	Wine	Wine	Wine	Wine	Wine

seq	size	emb **(start:end)**		
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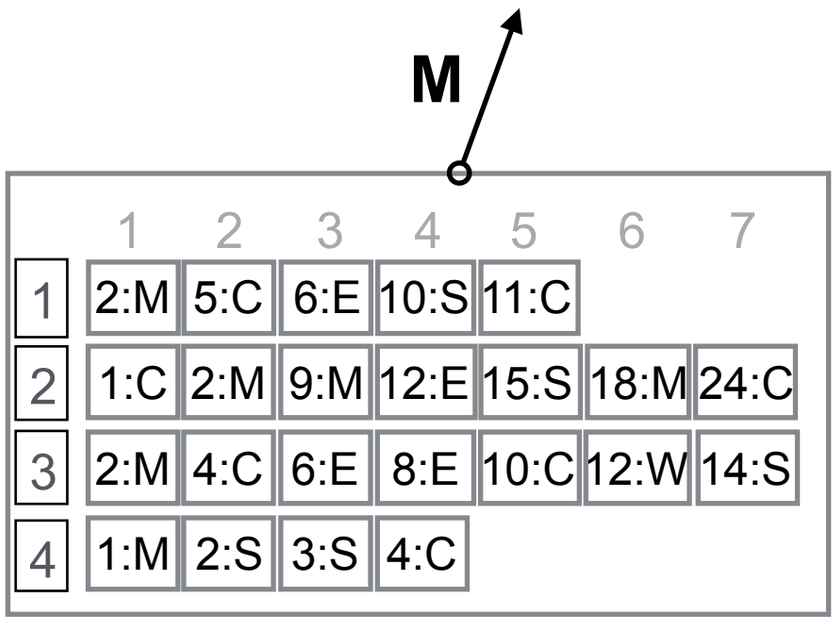
MinSup=3
Gap[3,7] t.u.



Supports
M : 4
C : 4
S : 4
E : 3
~~W : 1~~

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg	Egg	Egg
		Wine	Wine	Wine	Wine	Wine	Wine

seq	size	emb **(start:end)**		
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MinSup=3
Gap[3,7] t.u.

M

	1	2	3	4	5	6	7
1	2:M	5:C	**6:E**	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	2:M	4:C	6:E	8:E	10:C	**12:W**	14:S
4	1:M	2:S	3:S	4:C			

Supports
M : 4
C : 4
S : 4
E : 3
~~W : 1~~

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg	Egg	Egg
		Wine	Wine	Wine	Wine	Wine	Wine

seq	size	emb **(start:end)**		
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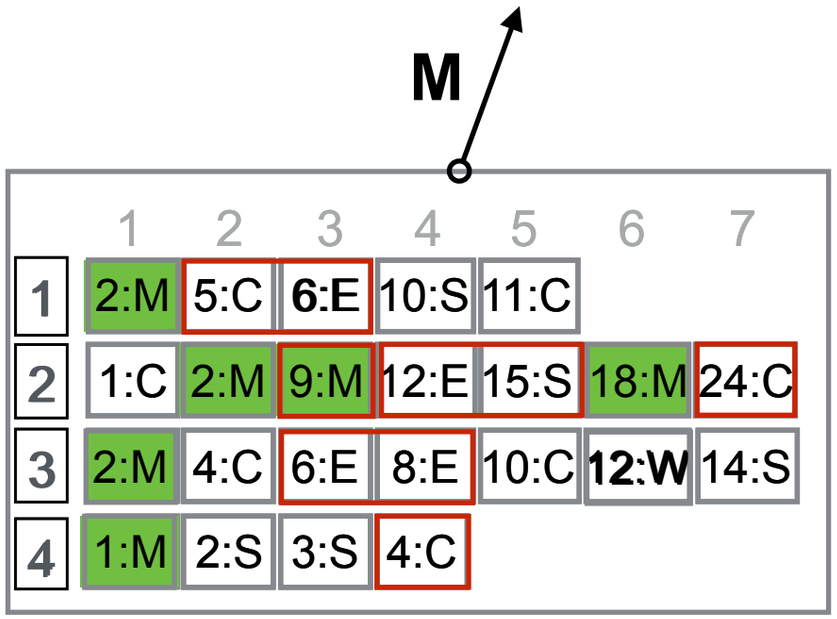
MinSup=3
Gap[3,7] t.u.



Supports
M : 4
C : 4
S : 4
E : 3
~~W : 1~~

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg	Egg	Egg
		Wine	Wine	Wine	Wine	Wine	Wine

seq	size	emb **(start:end)**		
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MinSup=3
Gap[3,7] t.u.

	1	2	3	4	5	6	7
1		5:C	6:E				
2			9:M	12:E	15:S		24:C
3			6:E	8:E			
4				4:C			

M

	1	2	3	4	5	6	7
1	2:M	5:C	**6:E**	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	2:M	4:C	6:E	8:E	10:C	**12:W**	14:S
4	1:M	2:S	3:S	4:C			

seq	size	emb **(start:end)**		
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Supports
M : 4
C : 4
S : 4
E : 3
~~W : 1~~

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg	Egg	Egg
		Wine	Wine	Wine	Wine	Wine	Wine

MinSup=3
Gap[3,7] t.u.

	1	2	3	4	5	6	7
1		5:C	6:E				
2			9:M	12:E	15:S		24:C
3			6:E	8:E			
4				4:C			

M

	1	2	3	4	5	6	7
1	2:M	5:C	**6:E**	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	2:M	4:C	6:E	8:E	10:C	**12:W**	14:S
4	1:M	2:S	3:S	4:C			

start=1

Size=4

seq	size	emb **(start:end)**		
1	1	1:1	.	.
2	3	2:2	3:3	6:6
3	1	1:1	.	.
4	1	1:1	.	.
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Supports
M : 4
C : 4
S : 4
E : 3
~~W : 1~~

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg	Egg	Egg
		Wine	Wine	Wine	Wine	Wine	Wine

MinSup=3
Gap[3,7] t.u.

~~M : 1~~
C : 3
~~S : 1~~
E : 3
~~W : 1~~

	1	2	3	4	5	6	7
1		5:C	6:E				
2			9:M	12:E	15:S		24:C
3			6:E	8:E			
4				4:C			

M

	1	2	3	4	5	6	7
1	2:M	5:C	**6:E**	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	2:M	4:C	6:E	8:E	10:C	**12:W**	14:S
4	1:M	2:S	3:S	4:C			

start=1

Size=4

seq	size	emb **(start:end)**		
1	1	1:1	.	.
2	3	2:2	3:3	6:6
3	1	1:1	.	.
4	1	1:1	.	.
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Supports
M : 4
C : 4
S : 4
E : 3
~~W : 1~~

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk		Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar		Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg	Egg	Egg
			Wine	Wine	Wine	Wine	Wine

MinSup=3
Gap[3,7] t.u.

~~M : 1~~
C : 3
~~S : 1~~
E : 3
~~W : 1~~

E

	1	2	3	4	5	6	7
1		5:C	6:E				
2			9:M	12:E	15:S		24:C
3			6:E	8:E			
4				4:C			

M

	1	2	3	4	5	6	7
1	2:M	5:C	**6:E**	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	2:M	4:C	6:E	8:E	10:C	**12:W**	14:S
4	1:M	2:S	3:S	4:C			

start=1

Size=4

seq	size	emb **(start:end)**		
1	1	1:1	.	.
2	3	2:2	3:3	6:6
3	1	1:1	.	.
4	1	1:1	.	.
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Supports
M : 4
C : 4
S : 4
E : 3
~~W : 1~~

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk		Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar		Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg	Egg	Egg
			Wine	Wine	Wine	Wine	Wine

MinSup=3
Gap[3,7] t.u.

~~M : 1~~
C : 3
~~S : 1~~
E : 3
~~W : 1~~

E

~~M : 1~~
~~C : 2~~
S : 3
~~E : 0~~
~~W : 1~~

	1	2	3	4	5	6	7
1		5:C	6:E				
2			9:M	12:E	15:S		24:C
3			6:E	8:E			
4				4:C			

	1	2	3	4	5	6	7
1				10:S	11:C		
2					15:S	18:M	
3					10:C	12:W	14:S

M

	1	2	3	4	5	6	7
1	2:M	5:C	6:E	10:S	11:C		
2	1:C	**2:M**	9:M	12:E	**15:S**	18:M	**24:C**
3	**2:M**	4:C	6:E	8:E	10:C	12:W	14:S
4	**1:M**	2:S	3:S	4:C			

Supports
M : 4
C : 4
S : 4
E : 3
~~W : 1~~

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di	ϵ	ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk			Milk	Milk	Milk	Milk
	Coffee	Coffee		Coffee	Coffee	Coffee	Coffee
	Sugar		Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg		Egg	Egg	Egg	Egg
				Wine	Wine	Wine	Wine

seq	size	emb **(start:end)**		
1	1	1:1	.	.
2	3	2:2	3:3	6:6
3	1	1:1	.	.
4	1	1:1	.	.
1	1	1:3	.	.
2	1	3:4	.	.
3	2	1:3	1:4	.
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start=5

Size=3

MinSup=3
Gap[3,7] t.u.

~~M : 1~~
C : 3
~~S : 1~~
E : 3
~~W : 1~~

E

~~M : 1~~
~~C : 2~~
S : 3
~~E : 0~~
~~W : 1~~

S

~~M : 1~~
~~C : 0~~
~~S : 0~~
~~E : 0~~
~~W : 0~~

	1	2	3	4	5	6	7
1		5:C	6:E				
2			9:M	12:E	15:S		24:C
3			6:E	8:E			
4				4:C			

	1	2	3	4	5	6	7
1					10:S	11:C	
2						15:S	18:M
3						10:C	12:W	14:S

	1	2	3	4	5	6	7
1							
2							18:M
3							

M

	1	2	3	4	5	6	7
1	2:M	5:C	**6:E**	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	**2:M**	4:C	6:E	8:E	10:C	12:W	14:S
4	**1:M**	2:S	3:S	4:C			

seq	size	emb **(start:end)**		
1	1	1:1	.	.
2	3	2:2	3:3	6:6
3	1	1:1	.	.
4	1	1:1	.	.
1	1	1:3	.	.
2	1	3:4	.	.
3	2	1:3	1:4	.
1	1	1:2	.	.
2	1	3:5	.	.
3	1	1:7	.	.
.	.	.	.	.
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start=8

Size=3

Supports
M : 4
C : 4
S : 4
E : 3
~~W : 1~~

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk			Milk	Milk	Milk	Milk
	Coffee	Coffee		Coffee	Coffee	Coffee	Coffee
	Sugar		Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg		Egg	Egg	Egg	Egg
				Wine	Wine	Wine	Wine

MinSup=3
Gap[3,7] t.u.

~~M : 1~~
C : 3
~~S : 1~~
E : 3
~~W : 1~~

E

~~M : 1~~
~~C : 2~~
S : 3
~~E : 0~~
~~W : 1~~

S

~~M : 1~~
~~C : 0~~
~~S : 0~~
~~E : 0~~
~~W : 0~~

	1	2	3	4	5	6	7
1		5:C	6:E				
2			9:M	12:E	15:S		24:C
3			6:E	8:E			
4				4:C			

	1	2	3	4	5	6	7
1					10:S	11:C	
2						15:S	18:M
3						10:C	12:W	14:S

	1	2	3	4	5	6	7
1							
2							18:M
3							

M

	1	2	3	4	5	6	7
1	2:M	5:C	**6:E**	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	**2:M**	4:C	6:E	8:E	10:C	12:W	14:S
4	**1:M**	2:S	3:S	4:C			

seq	size	emb **(start:end)**		
1	1	1:1	.	.
2	3	2:2	3:3	6:6
3	1	1:1	.	.
4	1	1:1	.	.
1	1	1:3	.	.
2	1	3:4	.	.
3	2	1:3	1:4	.
1	1	1:2	.	.
2	1	3:5	.	.
3	1	1:7	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

start=8

Size=3

Supports
M : 4
C : 4
S : 4
E : 3
~~W : 1~~

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk				Milk	Milk	Milk
	Coffee	Coffee			Coffee	Coffee	Coffee
	Sugar		Sugar		Sugar	Sugar	Sugar
	Egg	Egg			Egg	Egg	Egg
					Wine	Wine	Wine

MinSup=3
Gap[3,7] t.u.

~~M : 1~~
C : 3
~~S : 1~~
E : 3
~~W : 1~~

E

~~M : 1~~
~~C : 2~~
S : 3
~~E : 0~~
~~W : 1~~

S

~~M : 1~~
~~C : 0~~
~~S : 0~~
~~E : 0~~
~~W : 0~~

	1	2	3	4	5	6	7
1		5:C	6:E				
2			9:M	12:E	15:S		24:C
3			6:E	8:E			
4				4:C			

	1	2	3	4	5	6	7
1					10:S	11:C	
2						15:S	18:M
3						10:C	12:W	14:S

	1	2	3	4	5	6	7
1							
2							18:M
3							

M

	1	2	3	4	5	6	7
1	2:M	5:C	**6:E**	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	**2:M**	4:C	6:E	8:E	10:C	12:W	14:S
4	**1:M**	2:S	3:S	4:C			

seq	size	emb **(start:end)**		
1	1	1:1	.	.
2	3	2:2	3:3	6:6
3	1	1:1	.	.
4	1	1:1	.	.
1	1	1:3	.	.
2	1	3:4	.	.
3	2	1:3	1:4	.
1	1	1:2	.	.
2	1	3:5	.	.
3	1	1:7	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

start=8

Size=3

Supports
M : 4
C : 4
S : 4
E : 3
~~W : 1~~

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk						
	Coffee	Coffee					
	Sugar		Sugar				
	Egg	Egg					

Solution!

MinSup=3
Gap[3,7] t.u.

~~M : 1~~
C : 3
~~S : 1~~
E : 3
~~W : 1~~

E

~~M : 1~~
~~C : 2~~
S : 3
~~E : 0~~
~~W : 1~~

S

~~M : 1~~
~~C : 0~~
~~S : 0~~
~~E : 0~~
~~W : 0~~

	1	2	3	4	5	6	7
1		5:C	6:E				
2			9:M	12:E	15:S		24:C
3			6:E	8:E			
4				4:C			

	1	2	3	4	5	6	7
1					10:S	11:C	
2						15:S	18:M
3						10:C	12:W	14:S

	1	2	3	4	5	6	7
1							
2							18:M
3							

M

	1	2	3	4	5	6	7
1	2:M	5:C	**6:E**	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	**2:M**	4:C	6:E	8:E	10:C	12:W	14:S
4	**1:M**	2:S	3:S	4:C			

Backtrack

seq	size	emb **(start:end)**		
1	1	1:1	.	.
2	3	2:2	3:3	6:6
3	1	1:1	.	.
4	1	1:1	.	.
1	1	1:3	.	.
2	1	3:4	.	.
3	2	1:3	1:4	.
1	1	1:2	.	.
2	1	3:5	.	.
3	1	1:7	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

start=8

Size=3

Supports
M : 4
C : 4
S : 4
E : 3
~~W : 1~~

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
Di	Milk						
	Coffee	Coffee					
	Sugar		Sugar				
	Egg	Egg					

Solution!

MinSup=3
Gap[3,7] t.u.

~~M : 1~~
C : 3
~~S : 1~~
E : 3
~~W : 1~~

E

	1	2	3	4	5	6	7
1		5:C	6:E				
2			9:M	12:E	15:S		24:C
3			6:E	8:E			
4				4:C			

~~M : 1~~
~~C : 2~~
S : 3
~~E : 0~~
~~W : 1~~

S

	1	2	3	4	5	6	7
1				10:S	11:C		
2					15:S	18:M	
3					10:C	12:W	14:S

~~M : 1~~
~~C : 0~~
~~S : 0~~
~~E : 0~~
~~W : 0~~

	1	2	3	4	5	6	7
1							
2							18:M
3							

M

	1	2	3	4	5	6	7
1	**2:M**	5:C	6:E	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	2:M	4:C	**6:E**	8:E	10:C	12:W	14:S
4	**1:M**	2:S	**3:S**	4:C			

C

~~M : 0~~
~~C : 1~~
~~S : 1~~
~~E : 0~~
~~W : 0~~

	1	2	3	4	5	6	7
1				10:S	11:C		
2							
4							

Backtrack

seq	size	emb **(start:end)**		
1	1	1:1	.	.
2	3	2:2	3:3	6:6
3	1	1:1	.	.
4	1	1:1	.	.
1	1	1:2	.	.
2	1	6:7	.	.
4	1	1:4	.	.
1	1	1:2	.	.
2	1	3:5	.	.
3	1	1:7	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

start=5

Size=3

Supports
M : 4
C : 4
S : 4
E : 3
~~W : 1~~

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk		Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar		Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg	Egg	Egg
			Wine	Wine	Wine	Wine	Wine

MinSup=3
Gap[3,7] t.u.

~~M : 1~~
C : 3
~~S : 1~~
E : 3
~~W : 1~~

E

	1	2	3	4	5	6	7
1		5:C	6:E				
2			9:M	12:E	15:S		24:C
3			6:E	8:E			
4				4:C			

~~M : 1~~
~~C : 2~~
S : 3
~~E : 0~~
~~W : 1~~

S

	1	2	3	4	5	6	7
1				10:S	11:C		
2					15:S	18:M	
3					10:C	12:W	14:S

~~M : 1~~
~~C : 0~~
~~S : 0~~
~~E : 0~~
~~W : 0~~

	1	2	3	4	5	6	7
1							
2							18:M
3							

M

	1	2	3	4	5	6	7
1	**2:M**	5:C	6:E	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	2:M	4:C	**6:E**	8:E	10:C	12:W	14:S
4	**1:M**	2:S	**3:S**	4:C			

C

~~M : 0~~
~~C : 1~~
~~S : 1~~
~~E : 0~~
~~W : 0~~

	1	2	3	4	5	6	7
1				10:S	11:C		
2							
4							

Backtrack

seq	size	emb (**start:end**)		
1	1	1:1	.	.
2	3	2:2	3:3	6:6
3	1	1:1	.	.
4	1	1:1	.	.
1	1	1:2	.	.
2	1	6:7	.	.
4	1	1:4	.	.
1	1	1:2	.	.
2	1	3:5	.	.
3	1	1:7	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

start=5

Size=3

Supports
M : 4
C : 4
S : 4
E : 3
~~W : 1~~

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di		ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk						
	Coffee	Coffee					
	Sugar						
	Egg	Egg					

Solution!

EXPERIMENTS

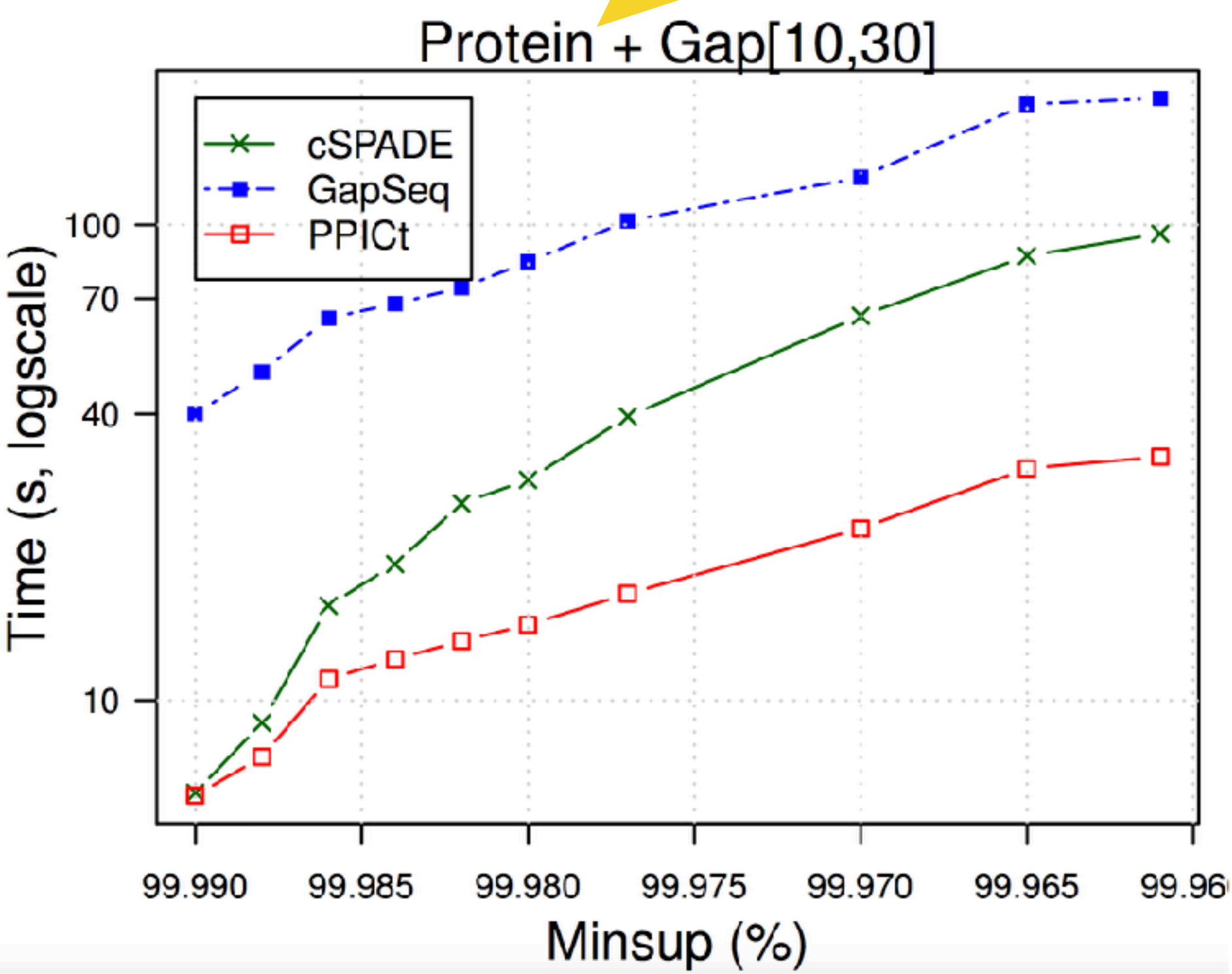
OSCAR Scala

www.oscarlib.org

COMPARED WITH EXISTING METHODS

Time limit = 3600s (1Hour)

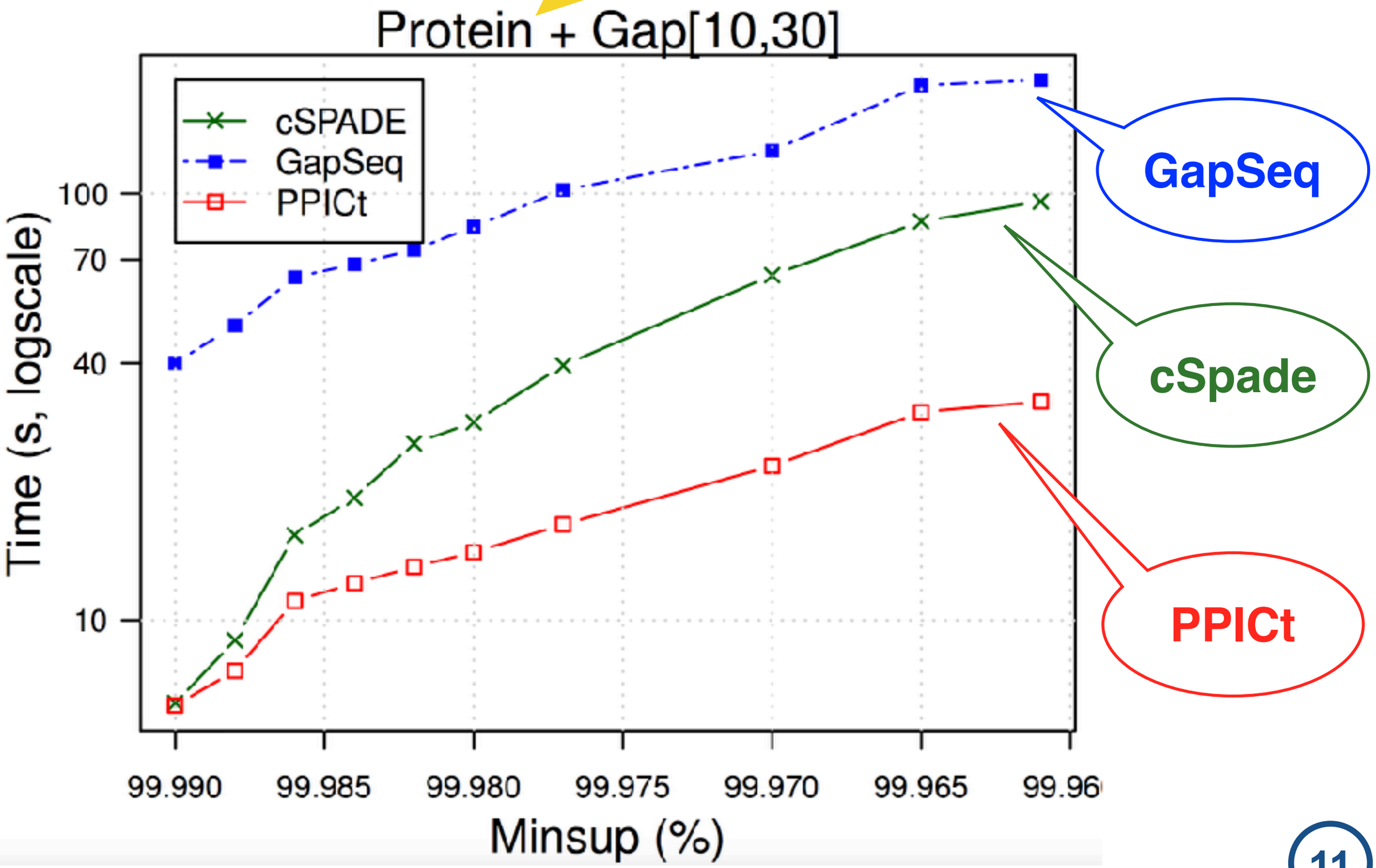
Largest and densest dataset (49,729,890 symbols) 600 variables



COMPARED WITH EXISTING METHODS

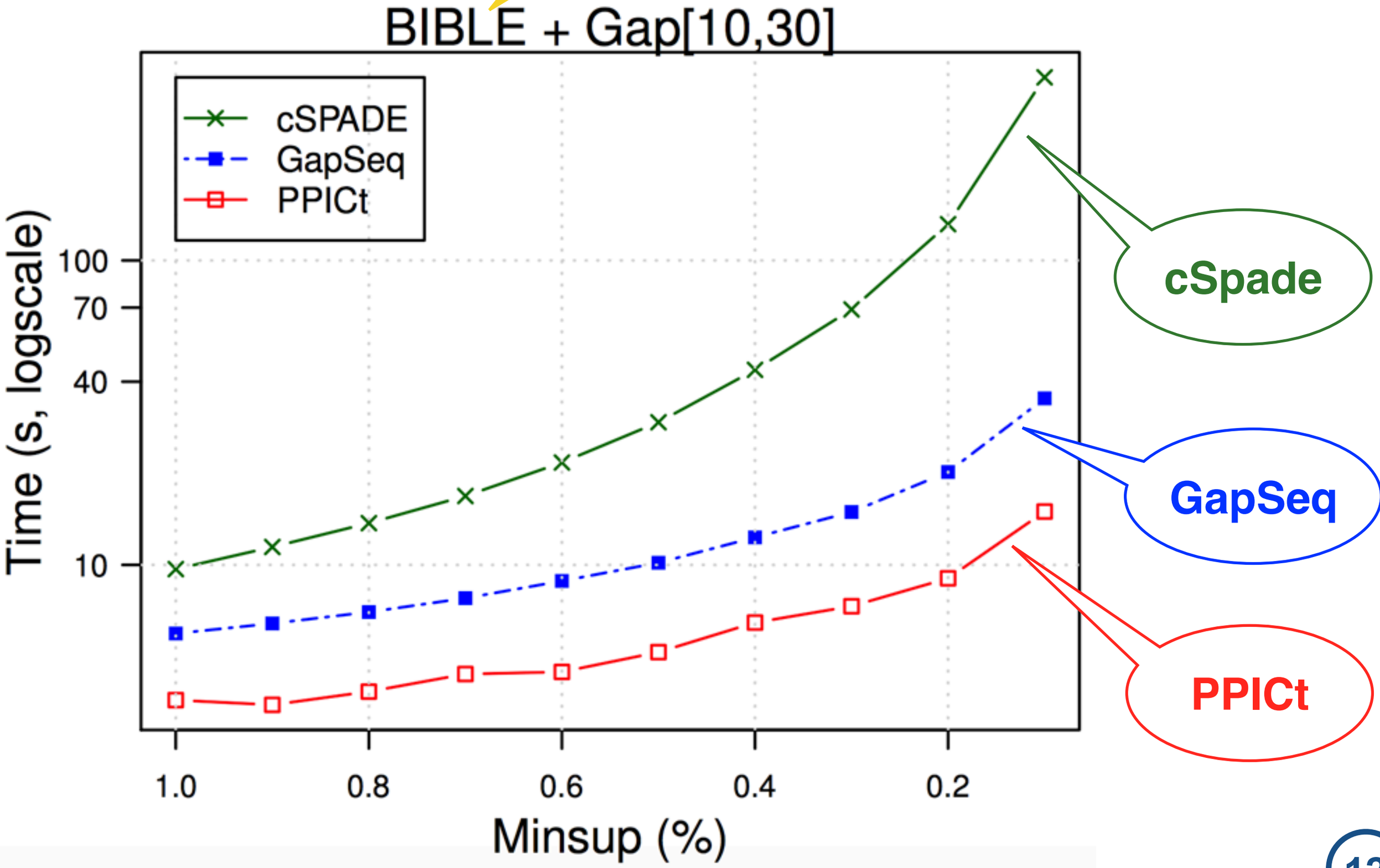
Time limit = 3600s (1Hour)

Largest and densest dataset (49,729,890 symbols) 600 variables



COMPARED WITH EXISTING METHODS

sparse dataset (787,066 symbols) 100 variables



Handling of different additional constraints

Methods	Frequency	Gap	Span	Regular/ Grammar	Among/ Gcc	Length
PPICt	x	x	x	x	x	x
GapSeq	x	x*			x	x
cSPADE	x	x	x**			x

Combining constraint over **Bible** dataset (13,905 symbols, 36,369 sequences)

+Gap

nSols : 32 307
Time(s) : 46

+Length+Gcc+Regular

nSols : 8
Time(s) : 0.19

Take-Away message

- Combining both SPM and CP techniques can lead to very efficient, modular and flexible approaches.

- Many kind of existing modules (in CP-Solvers) are reusable for free

- **Efficient memory using Trail-based backtracking aware data structure** really speed up search in DFSearch (not only for data mining)

- Code, data and apps are open **http://sites.uclouvain.be/cp4dm/spm/**

thank you!

	1	2	3	4	5	6	7
1	2:**M**	5:C	6:E	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	2:M	4:C	6:E	8:E	10:C	12:W	14:S
4	1:M	2:S	3:S	4:C			

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di	ϵ	ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg	Egg	Egg
	Wine	Wine	Wine	Wine	Wine	Wine	Wine

seq	size	emb **(start:end)**		
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	1	2	3	4	5	6	7
1	2:**M**	5:C	6:E	10:S	11:C		
2	1:C	2:M	9:M	12:E	15:S	18:M	24:C
3	2:M	4:C	6:E	8:E	10:C	12:W	14:S
4	1:M	2:S	3:S	4:C			

Vi	P_1	P_2	P_3	P_4	P_5	P_6	P_7
Di	ϵ	ϵ	ϵ	ϵ	ϵ	ϵ	ϵ
	Milk	Milk	Milk	Milk	Milk	Milk	Milk
	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee	Coffee
	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar	Sugar
	Egg	Egg	Egg	Egg	Egg	Egg	Egg
	Wine	Wine	Wine	Wine	Wine	Wine	Wine

seq	size	emb **(start:end)**		
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